Частинні похідні складених функцій. Частинні похідні і диференціали вищих порядків

Задача

Знайти повну похідну $\dfrac{\mathrm{d}z}{\mathrm{d}x}$, якщо $z = z\left(x, y\right)$, $y = y\left(x\right)$:

$$z = \frac{1}{\sqrt[3]{x^2 - y^3}} + \sin\frac{x}{y}; y = \operatorname{ctg}\frac{1}{x}.$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Маємо

$$\frac{\partial z}{\partial x} = -\frac{1}{3}(x^2 - y^3)^{-\frac{4}{3}} \cdot 2x + \cos\frac{x}{y} \cdot \frac{1}{y}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Маємо

$$\frac{\partial z}{\partial x} = -\frac{1}{3}(x^2 - y^3)^{-\frac{4}{3}} \cdot 2x + \cos\frac{x}{y} \cdot \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = \frac{1}{3}(x^2 - y^3)^{-\frac{4}{3}} \cdot 3y^2 + \cos\frac{x}{y} \cdot \left(-\frac{x}{y^2}\right)$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Маємо

$$\frac{\partial z}{\partial x} = -\frac{1}{3}(x^2 - y^3)^{-\frac{4}{3}} \cdot 2x + \cos\frac{x}{y} \cdot \frac{1}{y}$$

$$\frac{\partial z}{\partial y} = \frac{1}{3}(x^2 - y^3)^{-\frac{4}{3}} \cdot 3y^2 + \cos\frac{x}{y} \cdot \left(-\frac{x}{y^2}\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{\sin^2\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = -\frac{2}{3}(x^2 - y^3)^{-\frac{4}{3}} \cdot x + \cos\frac{x}{y} \cdot \frac{1}{y} + \left[(x^2 - y^3)^{-\frac{4}{3}} \cdot y^2 + \cos\frac{x}{y} \cdot \left(-\frac{x}{y^2}\right)\right] \frac{1}{x^2 \sin^2 \frac{1}{x}}$$

Задача

Знайти повну похідну $\dfrac{\mathrm{d}z}{\mathrm{d}x}$, якщо $z = z\left(x, y\right)$, $y = y\left(x\right)$:
$z = \arccos \dfrac{x}{\sin y} + \sqrt{\dfrac{x}{y}}; y = \dfrac{1}{\ln^2 2^x}$.

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Маємо

$$\frac{\partial z}{\partial x} = -\frac{1}{\sqrt{1 - \left(\frac{x}{\sin y}\right)^2}} \cdot \frac{1}{\sin y} + \sqrt{\frac{1}{y}} \cdot \frac{1}{2\sqrt{x}}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Маємо

$$\frac{\partial z}{\partial x} = -\frac{1}{\sqrt{1-\left(\frac{x}{\sin y}\right)^2}} \cdot \frac{1}{\sin y} + \sqrt{\frac{1}{y}} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{\partial z}{\partial y} = -\frac{1}{\sqrt{1-\left(\frac{x}{\sin y}\right)^2}} \cdot x \cdot \left(-\frac{1}{\sin^2 y}\right) \cdot \cos y + \sqrt{x} \cdot \left(-\frac{1}{2}y^{-\frac{3}{2}}\right)$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}$$

Маємо

$$\frac{\partial z}{\partial x} = -\frac{1}{\sqrt{1-\left(\frac{x}{\sin y}\right)^2}} \cdot \frac{1}{\sin y} + \sqrt{\frac{1}{y}} \cdot \frac{1}{2\sqrt{x}}$$

$$\frac{\partial z}{\partial y} = -\frac{1}{\sqrt{1-\left(\frac{x}{\sin y}\right)^2}} \cdot x \cdot \left(-\frac{1}{\sin^2 y}\right) \cdot \cos y + \sqrt{x} \cdot \left(-\frac{1}{2}y^{-\frac{3}{2}}\right)$$

$$\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{2}{x^3 \ln^2 2}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}x} = -\frac{1}{\sqrt{\sin^2 y - x^2}} + \frac{1}{2\sqrt{xy}} + \left[-\frac{x\cos y}{\sin y\sqrt{\sin^2 y - x^2}} - \frac{1}{2y}\sqrt{\frac{x}{y}}\right] \cdot \left(-\frac{2}{x^3 \ln^2 2}\right)$$

Задача

Функцію $y = y(x)$ задано рівнянням

$$x^2 - 3xy + 4y^2 - 2x + 3y + 2 = 0.$$

Знайти y'.

Розв'язання

Нехай функцію задано неявним чином:

$$F(x, y) = 0.$$

Розв'язання

Нехай функцію задано неявним чином:

$$F(x, y) = 0.$$

Тоді

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

Розв’язання

Нехай функцію задано неявним чином:

$$F(x, y) = 0.$$

Тоді

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

Звідки

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

Розв'язання

Нехай функцію задано неявним чином:

$$F(x,y)=0.$$

Тоді

$$\frac{\mathrm{d}F}{\mathrm{d}x}=\frac{\partial F}{\partial x}+\frac{\partial F}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x}=0$$

Звідки

$$y'=\frac{\mathrm{d}y}{\mathrm{d}x}=-\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

Маємо

$$\frac{\partial F}{\partial x}=2x-3y-2;$$

Розв'язання

Нехай функцію задано неявним чином:

$$F(x, y) = 0.$$

Тоді

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

Звідки

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

Маємо

$$\frac{\partial F}{\partial x} = 2x - 3y - 2; \frac{\partial F}{\partial y} = -3x + 8y + 3.$$

Розв'язання

Нехай функцію задано неявним чином:

$$F(x, y) = 0.$$

Тоді

$$\frac{\mathrm{d}F}{\mathrm{d}x} = \frac{\partial F}{\partial x} + \frac{\partial F}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

Звідки

$$y' = \frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{\frac{\partial F}{\partial x}}{\frac{\partial F}{\partial y}}$$

Маємо

$$\frac{\partial F}{\partial x} = 2x - 3y - 2; \frac{\partial F}{\partial y} = -3x + 8y + 3.$$

Отже

$$y' = -\frac{2x - 3y - 2}{-3x + 8y + 3}.$$

Задача

Функцію $y = y(x)$ задано рівнянням

$$x^2 + 2xy + y^2 - 4x + 2y - 2 = 0.$$

Знайти y'.

Розв'язання

Розв'язати самостійно.

Задача

Знайти $\frac{\mathrm{d}z}{\mathrm{d}t}$, якщо $z = z\left(x, y\right)$, $x = x\left(t\right)$, $y = y\left(t\right)$:
$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2$.

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1+2x^2)e^{x^2+y^2+1}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1+2x^2)e^{x^2+y^2+1}$$

$$\frac{\partial z}{\partial y} = xe^{x^2+y^2+1} \cdot 2y$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1+2x^2)e^{x^2+y^2+1}$$

$$\frac{\partial z}{\partial y} = xe^{x^2+y^2+1} \cdot 2y = 2xye^{x^2+y^2+1}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1 + 2x^2)e^{x^2+y^2+1}$$

$$\frac{\partial z}{\partial y} = xe^{x^2+y^2+1} \cdot 2y = 2xye^{x^2+y^2+1}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2\cos t \sin t$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1+2x^2)e^{x^2+y^2+1}$$

$$\frac{\partial z}{\partial y} = xe^{x^2+y^2+1} \cdot 2y = 2xye^{x^2+y^2+1}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2\cos t \sin t = -\sin 2t$$

Розв’язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1+2x^2)e^{x^2+y^2+1}$$

$$\frac{\partial z}{\partial y} = xe^{x^2+y^2+1} \cdot 2y = 2xye^{x^2+y^2+1}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2\cos t \sin t = -\sin 2t$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\sin t^2 \cdot 2t$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = xe^{x^2+y^2+1}; x = \cos^2 t; y = \cos t^2.$$

$$\frac{\partial z}{\partial x} = e^{x^2+y^2+1} + xe^{x^2+y^2+1} \cdot 2x = (1+2x^2)e^{x^2+y^2+1}$$

$$\frac{\partial z}{\partial y} = xe^{x^2+y^2+1} \cdot 2y = 2xye^{x^2+y^2+1}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2\cos t \sin t = -\sin 2t$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\sin t^2 \cdot 2t = -2t\sin t^2$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -(1+2x^2)e^{x^2+y^2+1}\sin 2t - 2xye^{x^2+y^2+1}\cdot 2t\sin t^2$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -(1+2x^2)e^{x^2+y^2+1}\sin 2t - 2xye^{x^2+y^2+1}\cdot 2t\sin t^2 =$$

$$= -(1+2\cos^4 t)e^{\cos^4 t+\cos^2 t^2+1}\sin 2t - 2\cos^2 t\cos t^2 e^{\cos^4 t+\cos^2 t^2+1} 2t\sin t^2$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = -(1+2x^2)e^{x^2+y^2+1}\sin 2t - 2xye^{x^2+y^2+1}\cdot 2t\sin t^2 =$$

$$= -(1+2\cos^4 t)e^{\cos^4 t+\cos^2 t^2+1}\sin 2t - 2\cos^2 t\cos t^2 e^{\cos^4 t+\cos^2 t^2+1} 2t\sin t^2 =$$

$$= -\sin 2t(1+2\cos^4 t)e^{\cos^4 t+\cos^2 t^2+1} - 2t\cos^2 t\sin 2t^2 e^{\cos^4 t+\cos^2 t^2+1}$$

Задача

Знайти $\dfrac{\mathrm{d}z}{\mathrm{d}t}$, якщо $z = z\left(x, y\right)$, $x = x\left(t\right)$, $y = y\left(t\right)$:
$z = \sqrt{2\ln x + e^y} + \dfrac{y}{\sin x}; x = \sqrt{\sin t}; y = \dfrac{1}{\sqrt{t}}$.

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x = \frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x = \frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot e^y + \frac{1}{\sin x}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x = \frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot e^y + \frac{1}{\sin x} = \frac{e^y}{2\sqrt{2\ln x + e^y}} + \frac{1}{\sin x}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x = \frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot e^y + \frac{1}{\sin x} = \frac{e^y}{2\sqrt{2\ln x + e^y}} + \frac{1}{\sin x}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2\sqrt{\sin t}} \cdot \cos t$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x = \frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot e^y + \frac{1}{\sin x} = \frac{e^y}{2\sqrt{2\ln x + e^y}} + \frac{1}{\sin x}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2\sqrt{\sin t}} \cdot \cos t = \frac{\cos t}{2\sqrt{\sin t}}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \frac{\partial z}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial z}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t}.$$

Маємо

$$z = \sqrt{2\ln x + e^y} + \frac{y}{\sin x}; x = \sqrt{\sin t}; y = \frac{1}{\sqrt{t}}$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot \frac{2}{x} - \frac{y}{\sin^2 x} \cdot \cos x = \frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}$$

$$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{2\ln x + e^y}} \cdot e^y + \frac{1}{\sin x} = \frac{e^y}{2\sqrt{2\ln x + e^y}} + \frac{1}{\sin x}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{1}{2\sqrt{\sin t}} \cdot \cos t = \frac{\cos t}{2\sqrt{\sin t}}$$

$$\frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{1}{2}t^{-\frac{3}{2}}$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \left[\frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}\right] \cdot \frac{\cos t}{2\sqrt{\sin t}} + \left[\frac{e^y}{2\sqrt{2\ln x + e^y}} + \frac{1}{\sin x}\right] \cdot \left(-\frac{1}{2}t^{-\frac{3}{2}}\right)$$

Розв'язання

$$\frac{\mathrm{d}z}{\mathrm{d}t} = \left[\frac{1}{x\sqrt{2\ln x + e^y}} - \frac{y\cos x}{\sin^2 x}\right]\cdot\frac{\cos t}{2\sqrt{\sin t}} + \left[\frac{e^y}{2\sqrt{2\ln x + e^y}} + \frac{1}{\sin x}\right]\cdot\left(-\frac{1}{2}t^{-\frac{3}{2}}\right) =$$

$$= \left[\frac{1}{\sqrt{\sin t}\sqrt{2\ln\sqrt{\sin t} + e^{\frac{1}{\sqrt{t}}}}} - \frac{\frac{1}{\sqrt{t}}\cos\sqrt{\sin t}}{\sin^2\sqrt{\sin t}}\right]\cdot\frac{\cos t}{2\sqrt{\sin t}} -$$

$$-\left[\frac{e^{\frac{1}{\sqrt{t}}}}{2\sqrt{2\ln\sqrt{\sin t} + e^{\frac{1}{\sqrt{t}}}}} + \frac{1}{\sin\sqrt{\sin t}}\right]\cdot\frac{1}{2}t^{-\frac{3}{2}}$$

Задача

Знайти $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$, де $z = z\,(x, y)$: $z = \operatorname{ctg}\dfrac{\sin x}{y} - x; x = u^2 v; y = 1 + u.$

Розв’язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Розв’язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = \operatorname{ctg}\frac{\sin x}{y} - x; x = u^2 v; y = 1 + u$$

Розв'язання

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u};\qquad \frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z=\operatorname{ctg}\frac{\sin x}{y}-x; x=u^2v; y=1+u$$

$$\frac{\partial z}{\partial x}=-\frac{1}{\sin^2\frac{\sin x}{y}}\cdot\frac{\cos x}{y}-1$$

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = \operatorname{ctg}\frac{\sin x}{y} - x; x = u^2 v; y = 1 + u$$

$$\frac{\partial z}{\partial x} = -\frac{1}{\sin^2\frac{\sin x}{y}} \cdot \frac{\cos x}{y} - 1 = -\frac{\cos x}{y\sin^2\frac{\sin x}{y}} - 1$$

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = \operatorname{ctg}\frac{\sin x}{y} - x; x = u^2 v; y = 1 + u$$

$$\frac{\partial z}{\partial x} = -\frac{1}{\sin^2 \frac{\sin x}{y}} \cdot \frac{\cos x}{y} - 1 = -\frac{\cos x}{y \sin^2 \frac{\sin x}{y}} - 1$$

$$\frac{\partial z}{\partial y} = -\frac{1}{\sin^2 \frac{\sin x}{y}} \cdot \left(-\frac{\sin x}{y^2}\right)$$

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = \operatorname{ctg}\frac{\sin x}{y} - x; x = u^2 v; y = 1 + u$$

$$\frac{\partial z}{\partial x} = -\frac{1}{\sin^2 \frac{\sin x}{y}} \cdot \frac{\cos x}{y} - 1 = -\frac{\cos x}{y \sin^2 \frac{\sin x}{y}} - 1$$

$$\frac{\partial z}{\partial y} = -\frac{1}{\sin^2 \frac{\sin x}{y}} \cdot \left(-\frac{\sin x}{y^2}\right) = \frac{\sin x}{y^2 \sin^2 \frac{\sin x}{y}}$$

Розв’язання

$$\frac{\partial x}{\partial u} = 2uv;$$

Розв'язання

$$\frac{\partial x}{\partial u} = 2uv; \frac{\partial x}{\partial v} = u^2;$$

Розв'язання

$$\frac{\partial x}{\partial u} = 2uv; \frac{\partial x}{\partial v} = u^2;$$

$$\frac{\partial y}{\partial u} = 1;$$

Розв'язання

$$\frac{\partial x}{\partial u} = 2uv; \frac{\partial x}{\partial v} = u^2;$$

$$\frac{\partial y}{\partial u} = 1; \frac{\partial y}{\partial v} = 0;$$

Розв'язання

$$\frac{\partial z}{\partial u} = \left[-\frac{\cos x}{y \sin^2 \frac{\sin x}{y}} - 1\right] \cdot 2uv + \left[\frac{\sin x}{y^2 \sin^2 \frac{\sin x}{y}}\right] \cdot 1$$

Розв'язання

$$\frac{\partial z}{\partial u} = \left[-\frac{\cos x}{y\sin^2\frac{\sin x}{y}} - 1\right]\cdot 2uv + \left[\frac{\sin x}{y^2\sin^2\frac{\sin x}{y}}\right]\cdot 1 =$$

$$= \left[-\frac{\cos u^2 v}{(1+u)\sin^2\frac{\sin u^2 v}{1+u}} - 1\right]\cdot 2uv + \frac{\sin u^2 v}{(1+u)^2\sin^2\frac{\sin u^2 v}{1+u}}$$

Розв'язання

$$\frac{\partial z}{\partial u} = \left[-\frac{\cos x}{y\sin^2\frac{\sin x}{y}} - 1\right]\cdot 2uv + \left[\frac{\sin x}{y^2\sin^2\frac{\sin x}{y}}\right]\cdot 1 =$$

$$= \left[-\frac{\cos u^2 v}{(1+u)\sin^2\frac{\sin u^2 v}{1+u}} - 1\right]\cdot 2uv + \frac{\sin u^2 v}{(1+u)^2\sin^2\frac{\sin u^2 v}{1+u}}$$

$$\frac{\partial z}{\partial v} = \left[-\frac{\cos x}{y\sin^2\frac{\sin x}{y}} - 1\right]\cdot u^2 + \left[\frac{\sin x}{y^2\sin^2\frac{\sin x}{y}}\right]\cdot 0$$

Розв'язання

$$\frac{\partial z}{\partial u} = \left[-\frac{\cos x}{y \sin^2 \frac{\sin x}{y}} - 1 \right] \cdot 2uv + \left[\frac{\sin x}{y^2 \sin^2 \frac{\sin x}{y}} \right] \cdot 1 =$$

$$= \left[-\frac{\cos u^2 v}{(1+u) \sin^2 \frac{\sin u^2 v}{1+u}} - 1 \right] \cdot 2uv + \frac{\sin u^2 v}{(1+u)^2 \sin^2 \frac{\sin u^2 v}{1+u}}$$

$$\frac{\partial z}{\partial v} = \left[-\frac{\cos x}{y \sin^2 \frac{\sin x}{y}} - 1 \right] \cdot u^2 + \left[\frac{\sin x}{y^2 \sin^2 \frac{\sin x}{y}} \right] \cdot 0 = \left[-\frac{\cos u^2 v}{(1+u) \sin^2 \frac{\sin u^2 v}{1+u}} - 1 \right] \cdot u^2$$

Задача

Знайти $\dfrac{\partial z}{\partial u}$, $\dfrac{\partial z}{\partial v}$, де $z = z\,(x, y)$: $z = 2^{x\cos y} + \operatorname{tg}\sqrt{y}$; $x = \operatorname{tg}\dfrac{u}{v}$; $y = \dfrac{1}{u}$.

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = 2^{x\cos y} + \operatorname{tg}\sqrt{y}; x = \operatorname{tg}\frac{u}{v}; y = \frac{1}{u}$$

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = 2^{x\cos y} + \operatorname{tg}\sqrt{y}; x = \operatorname{tg}\frac{u}{v}; y = \frac{1}{u}$$

$$\frac{\partial z}{\partial x} = 2^{x\cos y}\ln 2 \cdot \cos y$$

Розв'язання

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial u}; \qquad \frac{\partial z}{\partial v} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial v} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z = 2^{x\cos y} + \operatorname{tg}\sqrt{y}; x = \operatorname{tg}\frac{u}{v}; y = \frac{1}{u}$$

$$\frac{\partial z}{\partial x} = 2^{x\cos y}\ln 2 \cdot \cos y$$

$$\frac{\partial z}{\partial y} = 2^{x\cos y}\ln 2 \cdot (-x\sin y) + \frac{1}{\cos^2\sqrt{y}} \cdot \frac{1}{2\sqrt{y}}$$

Розв'язання

$$\frac{\partial z}{\partial u}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial u}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial u};\qquad \frac{\partial z}{\partial v}=\frac{\partial z}{\partial x}\frac{\partial x}{\partial v}+\frac{\partial z}{\partial y}\frac{\partial y}{\partial v}.$$

Маємо

$$z=2^{x\cos y}+\operatorname{tg}\sqrt{y}; x=\operatorname{tg}\frac{u}{v}; y=\frac{1}{u}$$

$$\frac{\partial z}{\partial x}=2^{x\cos y}\ln 2\cdot\cos y$$

$$\frac{\partial z}{\partial y}=2^{x\cos y}\ln 2\cdot(-x\sin y)+\frac{1}{\cos^2\sqrt{y}}\cdot\frac{1}{2\sqrt{y}}=-x\sin y\cdot 2^{x\cos y}\ln 2+\frac{1}{2\sqrt{y}\cos^2\sqrt{y}}$$

Розв'язання

$$\frac{\partial x}{\partial u} = \frac{1}{v \cos^2 \dfrac{u}{v}};$$

Розв'язання

$$\frac{\partial x}{\partial u} = \frac{1}{v\cos^2\dfrac{u}{v}}; \frac{\partial x}{\partial v} = -\frac{u}{v^2\cos^2\dfrac{u}{v}};$$

Розв'язання

$$\frac{\partial x}{\partial u} = \frac{1}{v\cos^2\dfrac{u}{v}}; \frac{\partial x}{\partial v} = -\frac{u}{v^2\cos^2\dfrac{u}{v}};$$

$$\frac{\partial y}{\partial u} = -\frac{1}{u^2};$$

Розв'язання

$$\frac{\partial x}{\partial u} = \frac{1}{v\cos^2\dfrac{u}{v}}; \frac{\partial x}{\partial v} = -\frac{u}{v^2\cos^2\dfrac{u}{v}};$$

$$\frac{\partial y}{\partial u} = -\frac{1}{u^2}; \frac{\partial y}{\partial v} = 0;$$

Розв'язання

$$\frac{\partial z}{\partial u} = [2^{x\cos y}\ln 2\cdot\cos y]\cdot\frac{1}{v\cos^2\dfrac{u}{v}} + \left[-x\sin y\cdot 2^{x\cos y}\ln 2 + \frac{1}{2\sqrt{y}\cos^2\sqrt{y}}\right]\cdot\left(-\frac{1}{u^2}\right)$$

Розв'язання

$$\frac{\partial z}{\partial u} = [2^{x\cos y}\ln 2\cdot\cos y]\cdot\frac{1}{v\cos^2\dfrac{u}{v}} + \left[-x\sin y\cdot 2^{x\cos y}\ln 2 + \frac{1}{2\sqrt{y}\cos^2\sqrt{y}}\right]\cdot\left(-\frac{1}{u^2}\right) =$$

$$= 2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\cdot\cos\frac{1}{u}\cdot\frac{1}{v\cos^2\frac{u}{v}} - \left[\frac{1}{2\sqrt{\frac{1}{u}}\cos^2\sqrt{\frac{1}{u}}} - \operatorname{tg}\frac{u}{v}\sin\frac{1}{u}\cdot 2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\right]\cdot\frac{1}{u^2}$$

Розв'язання

$$\frac{\partial z}{\partial u} = [2^{x\cos y}\ln 2\cdot\cos y]\cdot\frac{1}{v\cos^2\dfrac{u}{v}} + \left[-x\sin y\cdot 2^{x\cos y}\ln 2 + \frac{1}{2\sqrt{y}\cos^2\sqrt{y}}\right]\cdot\left(-\frac{1}{u^2}\right) =$$

$$= 2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\cdot\cos\frac{1}{u}\cdot\frac{1}{v\cos^2\frac{u}{v}} - \left[\frac{1}{2\sqrt{\frac{1}{u}}\cos^2\sqrt{\frac{1}{u}}} - \operatorname{tg}\frac{u}{v}\sin\frac{1}{u}\cdot 2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\right]\cdot\frac{1}{u^2}$$

$$\frac{\partial z}{\partial v} = [2^{x\cos y}\ln 2\cdot\cos y]\cdot\left(-\frac{u}{v^2\cos^2\dfrac{u}{v}}\right) + \left[-x\sin y\cdot 2^{x\cos y}\ln 2 + \frac{1}{2\sqrt{y}\cos^2\sqrt{y}}\right]\cdot 0$$

Розв'язання

$$\frac{\partial z}{\partial u} = [2^{x\cos y}\ln 2\cdot\cos y]\cdot\frac{1}{v\cos^2\dfrac{u}{v}} + \left[-x\sin y\cdot 2^{x\cos y}\ln 2 + \frac{1}{2\sqrt{y}\cos^2\sqrt{y}}\right]\cdot\left(-\frac{1}{u^2}\right) =$$

$$= 2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\cdot\cos\frac{1}{u}\cdot\frac{1}{v\cos^2\frac{u}{v}} - \left[\frac{1}{2\sqrt{\frac{1}{u}}\cos^2\sqrt{\frac{1}{u}}} - \operatorname{tg}\frac{u}{v}\sin\frac{1}{u}\cdot 2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\right]\cdot\frac{1}{u^2}$$

$$\frac{\partial z}{\partial v} = [2^{x\cos y}\ln 2\cdot\cos y]\cdot\left(-\frac{u}{v^2\cos^2\dfrac{u}{v}}\right) + \left[-x\sin y\cdot 2^{x\cos y}\ln 2 + \frac{1}{2\sqrt{y}\cos^2\sqrt{y}}\right]\cdot 0 =$$

$$= -2^{\operatorname{tg}\frac{u}{v}\cos\frac{1}{u}}\ln 2\cdot\cos\frac{1}{u}\cdot\frac{u}{v^2\cos^2\frac{u}{v}}$$

Задача

Знайти диференціали 1-го та 2-го порядків функції $z = \ln \sin \dfrac{x}{y}$.

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = \ln \sin \frac{x}{y}$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = \ln \sin \frac{x}{y}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\sin \frac{x}{y}} \cdot \cos \frac{x}{y} \cdot \frac{1}{y}$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = \ln\sin\frac{x}{y}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\sin\frac{x}{y}} \cdot \cos\frac{x}{y} \cdot \frac{1}{y} = \frac{\operatorname{ctg}\frac{x}{y}}{y}$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = \ln \sin \frac{x}{y}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\sin \frac{x}{y}} \cdot \cos \frac{x}{y} \cdot \frac{1}{y} = \frac{\operatorname{ctg} \frac{x}{y}}{y}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sin \frac{x}{y}} \cdot \cos \frac{x}{y} \cdot \left(-\frac{x}{y^2}\right)$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = \ln\sin\frac{x}{y}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\sin\frac{x}{y}} \cdot \cos\frac{x}{y} \cdot \frac{1}{y} = \frac{\operatorname{ctg}\frac{x}{y}}{y}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\sin\frac{x}{y}} \cdot \cos\frac{x}{y} \cdot \left(-\frac{x}{y^2}\right) = -\frac{x\operatorname{ctg}\frac{x}{y}}{y^2}$$

Розв'язання

Отже,

$$dz = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}dx - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}dy$$

Розв'язання

Отже,

$$dz = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}dx - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}dy = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(dx - \frac{x}{y}dy\right)$$

Розв'язання

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\mathrm{d}x - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg}\frac{x}{y}}{y}$$

Розв'язання

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\mathrm{d}x - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg}\frac{x}{y}}{y} = \frac{1}{y}\cdot\left(-\frac{1}{\sin^2\frac{x}{y}}\right)\cdot\frac{1}{y}$$

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\mathrm{d}x - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg}\frac{x}{y}}{y} = \frac{1}{y}\cdot\left(-\frac{1}{\sin^2\frac{x}{y}}\right)\cdot\frac{1}{y} = -\frac{1}{y^2\sin^2\frac{x}{y}}$$

Розв'язання

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\mathrm{d}x - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg}\frac{x}{y}}{y} = \frac{1}{y}\cdot\left(-\frac{1}{\sin^2\frac{x}{y}}\right)\cdot\frac{1}{y} = -\frac{1}{y^2\sin^2\frac{x}{y}}$$

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y}$$

Розв'язання

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg} \dfrac{x}{y}}{y}\mathrm{d}x - \frac{x \operatorname{ctg} \dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg} \dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg} \frac{x}{y}}{y} = \frac{1}{y}\cdot\left(-\frac{1}{\sin^2 \frac{x}{y}}\right)\cdot\frac{1}{y} = -\frac{1}{y^2 \sin^2 \frac{x}{y}}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{x \operatorname{ctg} \frac{x}{y}}{y^2}\right)$$

Розв'язання

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\mathrm{d}x - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg}\frac{x}{y}}{y} = \frac{1}{y}\cdot\left(-\frac{1}{\sin^2\frac{x}{y}}\right)\cdot\frac{1}{y} = -\frac{1}{y^2\sin^2\frac{x}{y}}$$

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{x\operatorname{ctg}\frac{x}{y}}{y^2}\right) = -\frac{1}{y^2}\left(\operatorname{ctg}\frac{x}{y} - x\frac{1}{\sin^2\frac{x}{y}}\cdot\frac{1}{y}\right)$$

Розв'язання

Отже,

$$\mathrm{d}z = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\mathrm{d}x - \frac{x\operatorname{ctg}\dfrac{x}{y}}{y^2}\mathrm{d}y = \frac{\operatorname{ctg}\dfrac{x}{y}}{y}\left(\mathrm{d}x - \frac{x}{y}\mathrm{d}y\right)$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\operatorname{ctg}\frac{x}{y}}{y} = \frac{1}{y}\cdot\left(-\frac{1}{\sin^2\frac{x}{y}}\right)\cdot\frac{1}{y} = -\frac{1}{y^2\sin^2\frac{x}{y}}$$

$$\frac{\partial^2 z}{\partial x\partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{x\operatorname{ctg}\frac{x}{y}}{y^2}\right) = -\frac{1}{y^2}\left(\operatorname{ctg}\frac{x}{y} - x\frac{1}{\sin^2\frac{x}{y}}\cdot\frac{1}{y}\right) =$$

$$= \frac{1}{y^2}\left(\frac{x}{y\sin^2\frac{x}{y}} - \operatorname{ctg}\frac{x}{y}\right)$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y}$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{x \operatorname{ctg} \frac{x}{y}}{y^2}\right)$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{x\operatorname{ctg}\frac{x}{y}}{y^2}\right) = -x\left(-\frac{2}{y^3}\operatorname{ctg}\frac{x}{y} - \frac{1}{y^2}\cdot\frac{1}{\sin^2\frac{x}{y}}\cdot\left[-\frac{x}{y^2}\right]\right)$$

Розв’язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{x \operatorname{ctg}\frac{x}{y}}{y^2}\right) = -x\left(-\frac{2}{y^3}\operatorname{ctg}\frac{x}{y} - \frac{1}{y^2}\cdot\frac{1}{\sin^2\frac{x}{y}}\cdot\left[-\frac{x}{y^2}\right]\right) =$$

$$= x\left(\frac{2}{y^3}\operatorname{ctg}\frac{x}{y} - \frac{x}{y^4\sin^2\frac{x}{y}}\right)$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{x\operatorname{ctg}\frac{x}{y}}{y^2}\right) = -x\left(-\frac{2}{y^3}\operatorname{ctg}\frac{x}{y} - \frac{1}{y^2}\cdot\frac{1}{\sin^2\frac{x}{y}}\cdot\left[-\frac{x}{y^2}\right]\right) =$$

$$= x\left(\frac{2}{y^3}\operatorname{ctg}\frac{x}{y} - \frac{x}{y^4\sin^2\frac{x}{y}}\right)$$

Таким чином,

$$\mathrm{d}^2 z = -\frac{(\mathrm{d}x)^2}{y^2\sin^2\frac{x}{y}} + \frac{2}{y^2}\left(\frac{x}{y\sin^2\frac{x}{y}} - \operatorname{ctg}\frac{x}{y}\right)\mathrm{d}x\mathrm{d}y + \left(\frac{2x}{y^3}\operatorname{ctg}\frac{x}{y} - \frac{x^2}{y^4\sin^2\frac{x}{y}}\right)(\mathrm{d}y)^2$$

Задача

Знайти диференціали 1-го та 2-го порядків функції $z = 2^{\ln(x-y)} + \cos^2 x$.

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Розв'язання

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$d^2 z = \frac{\partial^2 z}{\partial x^2} (dx)^2 + 2 \frac{\partial^2 z}{\partial x \partial y} dx dy + \frac{\partial^2 z}{\partial y^2} (dy)^2$$

Маємо

$$z = 2^{\ln(x-y)} + \cos^2 x$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = 2^{\ln(x-y)} + \cos^2 x$$

$$\frac{\partial z}{\partial x} = 2^{\ln(x-y)} \ln 2 \cdot \frac{1}{x-y} \cdot 1 + 2\cos x \cdot \sin x$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = 2^{\ln(x-y)} + \cos^2 x$$

$$\frac{\partial z}{\partial x} = 2^{\ln(x-y)} \ln 2 \cdot \frac{1}{x-y} \cdot 1 + 2\cos x \cdot \sin x = \frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

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Маємо

$$z = 2^{\ln(x-y)} + \cos^2 x$$

$$\frac{\partial z}{\partial x} = 2^{\ln(x-y)}\ln 2 \cdot \frac{1}{x-y} \cdot 1 + 2\cos x \cdot \sin x = \frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x$$

$$\frac{\partial z}{\partial y} = 2^{\ln(x-y)}\ln 2 \cdot \frac{1}{x-y} \cdot (-1)$$

Розв'язання

$$\mathrm{d}z = \frac{\partial z}{\partial x}\mathrm{d}x + \frac{\partial z}{\partial y}\mathrm{d}y$$

$$\mathrm{d}^2 z = \frac{\partial^2 z}{\partial x^2}(\mathrm{d}x)^2 + 2\frac{\partial^2 z}{\partial x \partial y}\mathrm{d}x\mathrm{d}y + \frac{\partial^2 z}{\partial y^2}(\mathrm{d}y)^2$$

Маємо

$$z = 2^{\ln(x-y)} + \cos^2 x$$

$$\frac{\partial z}{\partial x} = 2^{\ln(x-y)}\ln 2 \cdot \frac{1}{x-y} \cdot 1 + 2\cos x \cdot \sin x = \frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x$$

$$\frac{\partial z}{\partial y} = 2^{\ln(x-y)}\ln 2 \cdot \frac{1}{x-y} \cdot (-1) = -\frac{2^{\ln(x-y)}\ln 2}{x-y}$$

Розв'язання

Отже,

$$dz = \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) dx - \frac{2^{\ln(x-y)} \ln 2}{x-y} dy$$

Розв’язання

Отже,

$$\mathrm{d}z = \left(\frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x\right)\mathrm{d}x - \frac{2^{\ln(x-y)}\ln 2}{x-y}\mathrm{d}y$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial z}{\partial x}$$

Розв'язання

Отже,

$$\mathrm{d}z = \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) \mathrm{d}x - \frac{2^{\ln(x-y)} \ln 2}{x-y} \mathrm{d}y$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right)$$

Розв'язання

Отже,

$$\mathrm{d}z = \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) \mathrm{d}x - \frac{2^{\ln(x-y)} \ln 2}{x-y} \mathrm{d}y$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) = \frac{2^{\ln(x-y)} \ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)} \ln 2}{(x-y)^2} + 2 \cos 2x$$

Розв'язання

Отже,

$$\mathrm{d}z = \left(\frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x\right)\mathrm{d}x - \frac{2^{\ln(x-y)}\ln 2}{x-y}\mathrm{d}y$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x\right) = \frac{2^{\ln(x-y)}\ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)}\ln 2}{(x-y)^2} + 2\cos 2x$$

$$= \frac{2^{\ln(x-y)}\ln 2(\ln 2 - 1)}{(x-y)^2} + 2\cos 2x$$

Розв'язання

Отже,

$$dz = \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) dx - \frac{2^{\ln(x-y)} \ln 2}{x-y} dy$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) = \frac{2^{\ln(x-y)} \ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)} \ln 2}{(x-y)^2} + 2\cos 2x$$

$$= \frac{2^{\ln(x-y)} \ln 2(\ln 2 - 1)}{(x-y)^2} + 2\cos 2x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \frac{\partial z}{\partial y}$$

Розв'язання

Отже,

$$\mathrm{d}z = \left(\frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x\right)\mathrm{d}x - \frac{2^{\ln(x-y)}\ln 2}{x-y}\mathrm{d}y$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\frac{2^{\ln(x-y)}\ln 2}{x-y} + \sin 2x\right) = \frac{2^{\ln(x-y)}\ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)}\ln 2}{(x-y)^2} + 2\cos 2x$$

$$= \frac{2^{\ln(x-y)}\ln 2(\ln 2 - 1)}{(x-y)^2} + 2\cos 2x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{2^{\ln(x-y)}\ln 2}{x-y}\right)$$

Розв'язання

Отже,

$$dz = \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) dx - \frac{2^{\ln(x-y)} \ln 2}{x-y} dy$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\frac{\partial z}{\partial x} = \frac{\partial}{\partial x}\left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) = \frac{2^{\ln(x-y)} \ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)} \ln 2}{(x-y)^2} + 2\cos 2x$$

$$= \frac{2^{\ln(x-y)} \ln 2(\ln 2 - 1)}{(x-y)^2} + 2\cos 2x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\frac{\partial z}{\partial y} = \frac{\partial}{\partial x}\left(-\frac{2^{\ln(x-y)} \ln 2}{x-y} \right) = -\frac{2^{\ln(x-y)} \ln^2 2}{(x-y)^2} + \frac{2^{\ln(x-y)} \ln 2}{(x-y)^2}$$

Розв'язання

Отже,

$$dz = \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) dx - \frac{2^{\ln(x-y)} \ln 2}{x-y} dy$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x} \frac{\partial z}{\partial x} = \frac{\partial}{\partial x} \left(\frac{2^{\ln(x-y)} \ln 2}{x-y} + \sin 2x \right) = \frac{2^{\ln(x-y)} \ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)} \ln 2}{(x-y)^2} + 2\cos 2x$$

$$= \frac{2^{\ln(x-y)} \ln 2(\ln 2 - 1)}{(x-y)^2} + 2\cos 2x$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x} \frac{\partial z}{\partial y} = \frac{\partial}{\partial x} \left(-\frac{2^{\ln(x-y)} \ln 2}{x-y} \right) = -\frac{2^{\ln(x-y)} \ln^2 2}{(x-y)^2} + \frac{2^{\ln(x-y)} \ln 2}{(x-y)^2} =$$

$$= \frac{2^{\ln(x-y)} \ln 2(1 - \ln 2)}{(x-y)^2}$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y}$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{2^{\ln(x-y)}\ln 2}{x-y}\right)$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{2^{\ln(x-y)}\ln 2}{x-y}\right) = \frac{2^{\ln(x-y)}\ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)}\ln 2}{(x-y)^2}$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{2^{\ln(x-y)}\ln 2}{x-y}\right) = \frac{2^{\ln(x-y)}\ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)}\ln 2}{(x-y)^2} =$$

$$= \frac{2^{\ln(x-y)}\ln 2(\ln 2 - 1)}{(x-y)^2}$$

Розв'язання

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\frac{\partial z}{\partial y} = \frac{\partial}{\partial y}\left(-\frac{2^{\ln(x-y)}\ln 2}{x-y}\right) = \frac{2^{\ln(x-y)}\ln^2 2}{(x-y)^2} - \frac{2^{\ln(x-y)}\ln 2}{(x-y)^2} =$$

$$= \frac{2^{\ln(x-y)}\ln 2(\ln 2 - 1)}{(x-y)^2}$$

Таким чином,

$$d^2 z =$$

$$= \left(\frac{2^{\ln(x-y)}\ln 2(\ln 2 - 1)}{(x-y)^2} + 2\cos 2x\right)(dx)^2 + \frac{2^{\ln(x-y)}\ln 2(1-\ln 2)}{(x-y)^2}\left(dx dy - (dy)^2\right)$$