THE MEDIAN OF TWO SEQUENCES

COURSE WORK #08

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PROBLEM STATEMENT

You have a sequence generated by a simple formula: $X_i = A*i + C$ where **A** and **C** are some constants. If **A** and **C** are positive integer numbers then the sequence is sorted in ascending order. Your task is to take two of such sequences with different constants and different sizes and find the median of the merged sorted sequence.

Input. The first string contains three integer numbers: A_1 ($0 <= A_1 <= 10^9$), C_1 ($0 <= C_1 <= 10^9$) and N_1 ($1 <= N_1 <= 10^9$). The second string contains three integer numbers: A_2 ($0 <= A_2 <= 10^9$), C_2 ($0 <= C_2 <= 10^9$) and N_2 ($1 <= N_2 <= 10^9$).

Output. The single number — the median of the merged sequence.

PROBLEM STATEMENT

Examples

Input:

1 1 2 [1*0+1, 1*1+1] = [1, 2]

1 3 2 [1*0+3, 1*1+4] = [3, 4]

Output: 2.50 [1,2] + [3,4] = [1,2,3,4].sort() = [1, 2, 3, 4]

Input:

7 13 7 [7*0+13, 7*1+13,...] = [13, 20, 27, 34, 41, 48, 55]

1 22 6 [1*0+22, 1*1+22,...] = [22, 23, 24, 25, 26, 27]

Output: 26.00 [13, 20, 22, 23, 24, 25, 26, 27, 27, 34, 41, 48, 55]

THE PROBLEM SOLVING

The simplest method to solve this problem is to define an array with size N_1+N_2 and store generated sequences inside this array. Then sort the array and find the median. This solution is simple, but according to the problem statement we have the maximum size of both sequences 10^9 and this is a huge amount of memory (2*3,7 GB) to store this data.

Thus, we apply another approach without an auxiliary array. As we know the sizes of both sequences, we can compute the median element positions — if the sum N_1+N_2 is even we need two indexes and one index otherwise. Then we will generate elements of both sequences in ascending order until we reach the median index.

THE PROBLEM SOLVING

The first example. The merged sequence size is (N_1+N_2)=(2+2)=4.

1. Generate the first element of the first sequence: 1.
2. Generate the first element of the second sequence: 3.
3. Compare two elements and choose the lesser: 1 < 3 so 1 is the first element of the merged sequence.
4. As the first element is chosen from the first sequence, the index of the first sequence increases.

THE PROBLEM SOLVING

5. Generate the second element of the first sequence: 2.
6. Compare the second element from the first sequence with the first element of the second sequence and choose the lesser: $2 < 3$ so 2 is the second element of the merged sequence.
7. As the second element is chosen from the first sequence, the index of the first sequence increases.
8. The index of the first sequence reached the end of the sequence, so we can not generate any elements from the first sequence.

1	2		

THE PROBLEM SOLVING

9. Add the first generated element from the second sequence to the merged sequence.
10. The current index of the merged sequence is $(N_1+N_2)/2+1$, so we already have both middle elements to compute the median.
11. Calculate the median of the merged sequence as the sum of the two last elements in the merged sequence divided by two: (2+3)/2=2.50.

As you can see, the first element of the merged sequence is not needed for the median computation, so we do not have to store it.

1	2	3	

THE PROBLEM SOLVING

The second example. The merged sequence size is (N_1+N_2)=(7+6)=13. But as we know from the previous example, we need only half size: 13/2+1=7 elements in the merged sequence.

1. Generate the first element of the first sequence: 13.
2. Generate the first element of the second sequence: 22.
3. Compare two elements and choose the lesser: 13 < 22 so 13 is the first element of the merged sequence.
4. As the first element is chosen from the first sequence, the index of the first sequence increases.

13						

THE PROBLEM SOLVING

5. Generate the second element of the first sequence: 20.
6. Compare the second element from the first sequence with the first element of the second sequence and choose the lesser: 20 < 22 so 20 is the second element of the merged sequence.
7. As the second element is chosen from the first sequence, the index of the first sequence increases.
8. Generate the third element of the first sequence: 27.
9. Compare the current elements from both sequences: 27 > 22 so 22 is the next element.

13	20	22				

THE PROBLEM SOLVING

10. Generate the second element of the second sequence: 23.
11. Compare the current elements from both sequences: 27 > 23 so 23 is the next element.
12. Generate the next element of the second sequence: 24.
13. Compare the current elements from both sequences: 27 > 24 so 24 is the next element.
14. Continue generating the second sequence until we reach the last element.

13	20	22	23	24	25	26

THE PROBLEM SOLVING

The algorithm to solve this problem.

1. Initialize two pointers i and j to move forward in both sequences.
2. Initialize three variables: index1 and index2 to keep track of the middle elements and count to track the count of elements in the merged array.
3. Start the merging loop and move forward by comparing elements in both sequences until the count becomes equal to $(N_1+N_2)/2+1$. We also keep track of middle elements index1 and index2.
4. By the end of the loop, we return the median by taking an average of index1 and index2.

THANK YOU!