NUMBER SEQUENCES

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WHAT IS A SEQUENCE?

A sequence is a list of numbers (or elements) that exhibit a particular pattern. Each element in the sequence is called a term. A sequence can be finite, meaning it has a specific number of terms, or infinite, meaning it continues indefinitely. Sequences can be described in different ways, such as an explicit formula, a recurrence relation, or a table of values.

- An explicit formula gives a direct way to compute each term in the sequence.
- A recurrence relation expresses each term in terms of one or more preceding terms.
- A table of values simply lists the terms in the sequence.

ORDER OF THE SEQUENCE

There are several types of sequences in math such as arithmetic sequences, quadratic sequences, geometric sequences, triangular sequences, square number sequences, cube number sequences, and triangular number sequences.

The order of a sequence can be either ascending or descending.

If the elements of the sequence are in increasing order, then the order of the sequence is ascending.

If the elements of the sequence are in decreasing order, then the order of the sequence is descending.

FINITE AND INFINITE SEQUENCES

There are two types of sequences: finite sequences and infinite sequences. It is possible to count the number of terms in a finite sequence. For example, a sequence of the number of bounces a ball takes to come to the rest is a finite sequence.

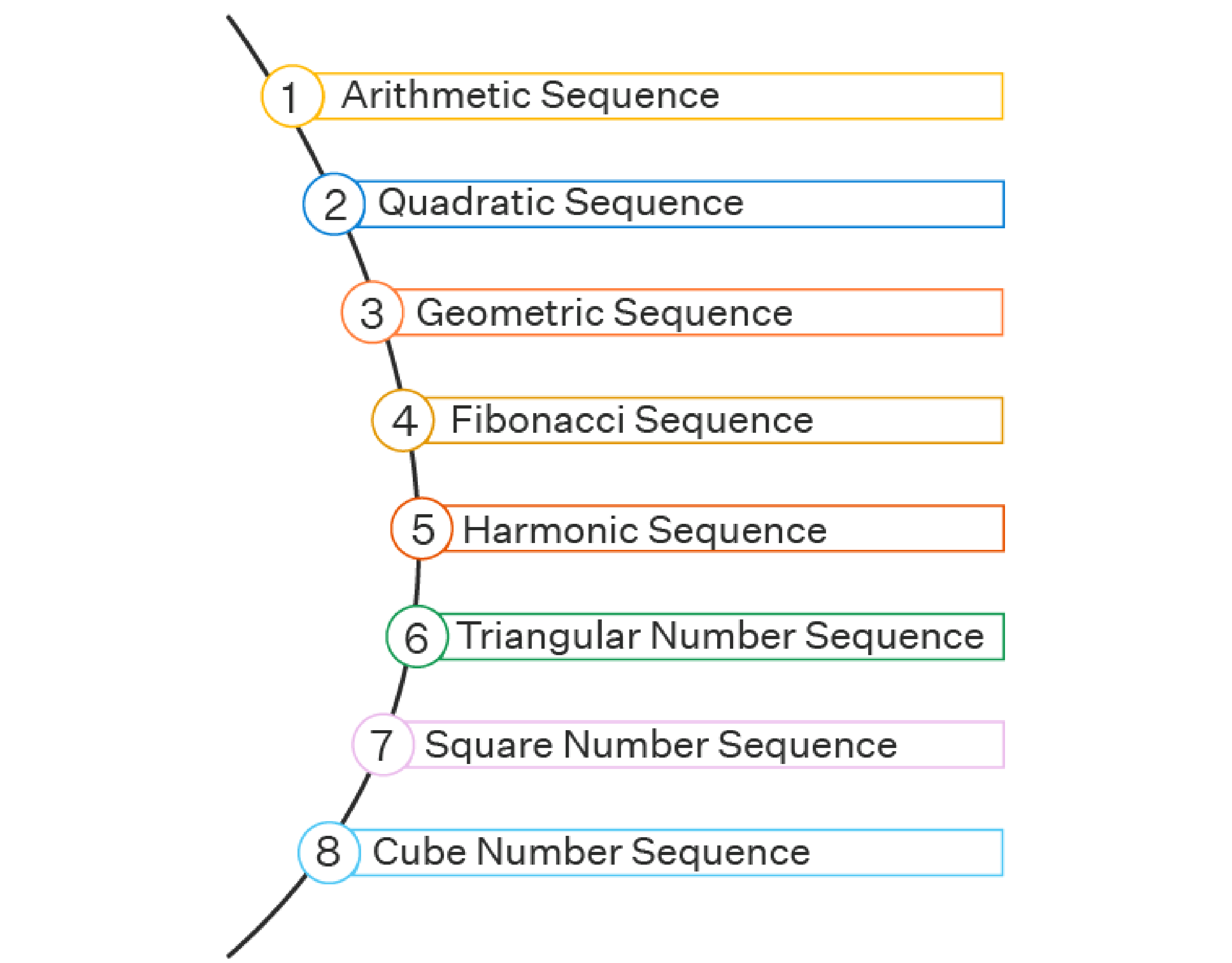
An infinite sequence is a sequence that is not finite.

For example, a sequence of natural numbers forms an infinite sequence: 1, 2, 3, 4, and so on.

TYPES OF SEQUENCES IN MATH

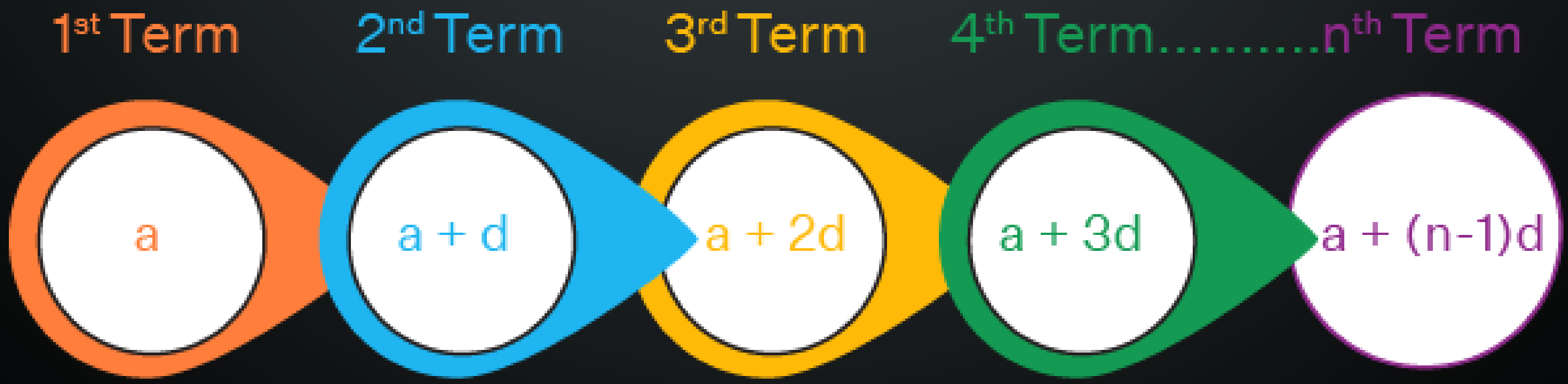
There are a few special sequences like arithmetic sequence, geometric sequence, Fibonacci sequence, harmonic sequence, triangular number sequence, square number sequence, and cube number sequence. Apart from these, there can be sequences that follow some other pattern. For example, 2, 9, 28, 65, ... is a sequence in which the numbers can be written as $1^3 + 1$, $2^3 + 1$, $3^3 + 1$, $4^3 + 1$,.... and this sequence does not belong to any of the following sequences.

TYPES OF SEQUENCES IN MATH



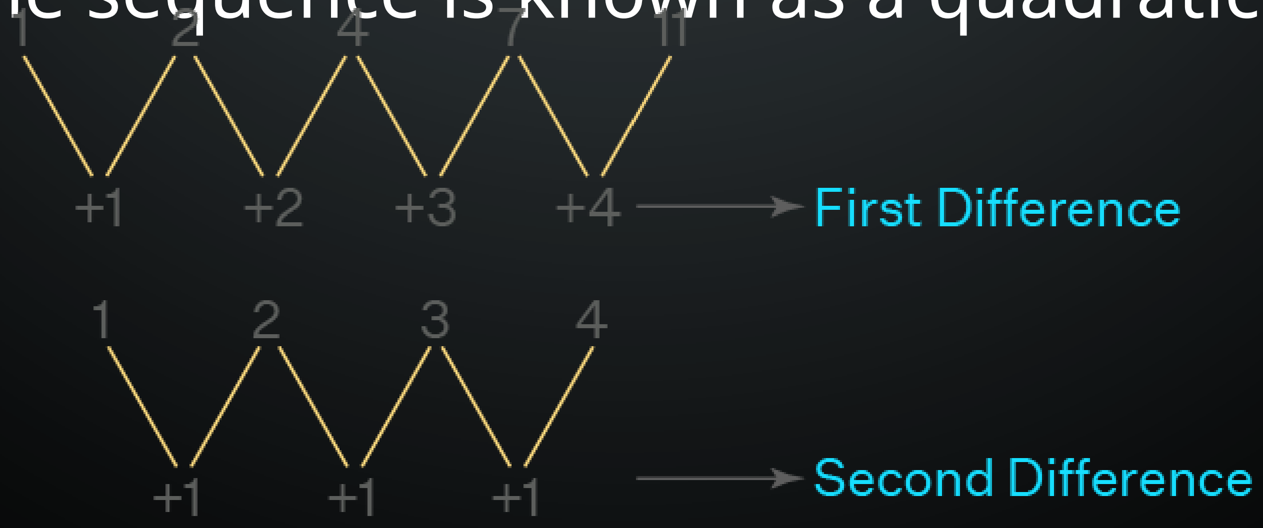
ARITHMETIC SEQUENCE

An arithmetic sequence is a sequence of numbers in which each successive term is a sum of its preceding term and a fixed number. This fixed number is called a common difference. The terms of the arithmetic sequence are of the form a, a+d, a+2d, ….



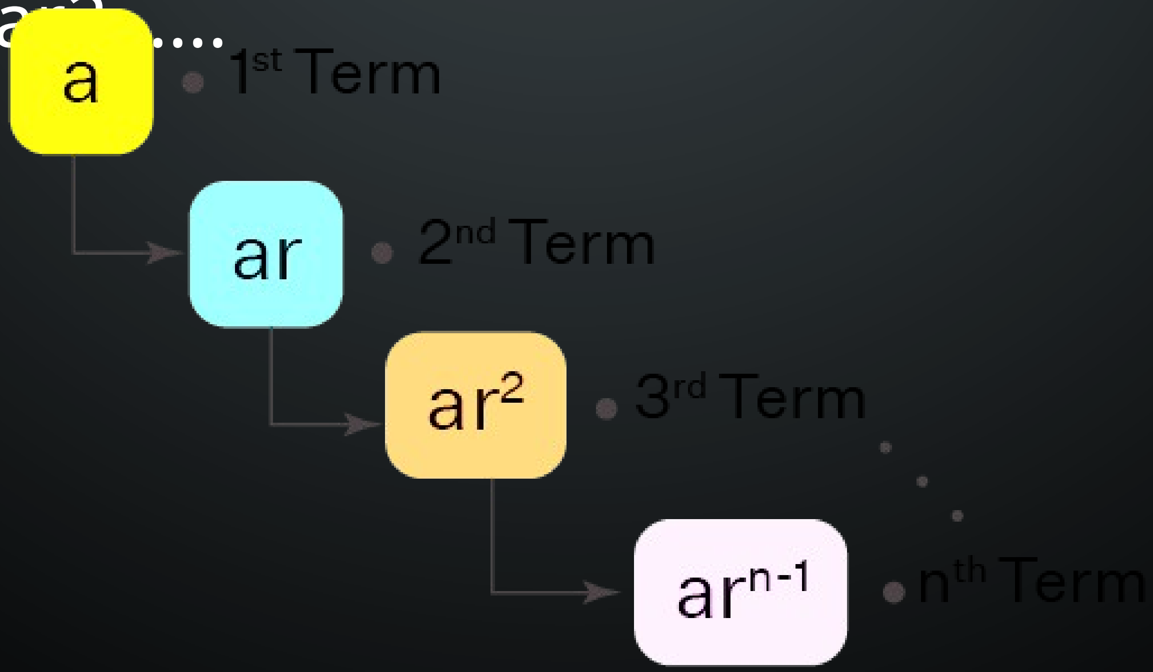
QUADRATIC SEQUENCE

We have already seen that if the differences (referred to as first differences) between every two successive terms are the same, then it is called an arithmetic sequence (which is also known as a linear sequence). But if the first differences are NOT the same, and instead, the second differences are the same, then the sequence is known as a quadratic sequence.



GEOMETRIC SEQUENCE

A geometric sequence is a sequence where every term bears a constant ratio to its preceding term. This ratio is called the "common ratio". The terms of the geometric sequence are of the form a, ar, ar^2, ...



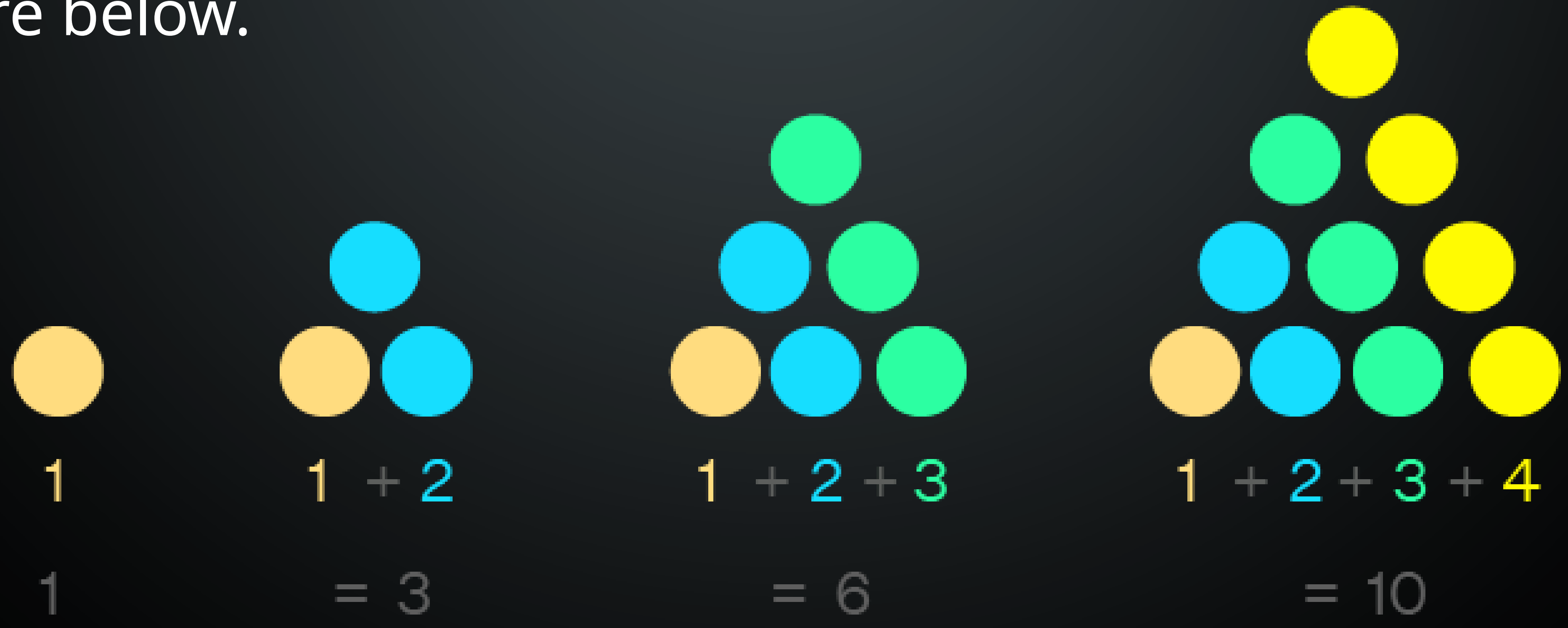
HARMONIC SEQUENCE

A harmonic sequence is a sequence obtained by taking the reciprocal of the terms of an arithmetic sequence.

Example: We know that the sequence of natural numbers is an arithmetic sequence. So, taking reciprocals of each term, we get 1, 1/2, 1/3, ..., which is a harmonic sequence as their reciprocals 1, 2, 3, ... form an arithmetic sequence.

TRIANGULAR NUMBER SEQUENCE

A triangular number sequence is a sequence that is obtained from a pattern forming equilateral triangles. Look at the figure below.



SQUARE NUMBER SEQUENCE

A square number sequence is a sequence that is obtained from a pattern forming squares. Look at the figure below.

CUBE NUMBER SEQUENCE

A cube number sequence is a sequence that is obtained from a pattern forming cubes. Look at the figure below.

FIBONACCI SEQUENCE

Fibonacci sequence is a sequence where every term is the sum of the last two preceding terms.

Example: A pair of rabbits do not reproduce in their 1st month. Starting from the 2nd month and every subsequent month, they reproduce another pair. Thus, the number of rabbits starting from 1st month is 0, 1, 1, 2, 3, 4, 7, 11, …. This is called the Fibonacci sequence.

SERIES AND PARTIAL SUMS OF SEQUENCES

Consider a sequence given by a_1, a_2, a_3, a_4, …. Then the sum $a_1 + a_2 + a_3 + a_4 + ...$ is the series associated with the sequence. Series can be represented using sigma notation, ∑. So, the series is represented as ∑n=1∞ a_n. The partial sum is a part of the series. The sum up to k terms in the series ∑n=1^k an and it is called the partial sum of the series.

Example: Consider a sequence of prime numbers: 2, 3, 5, 7, 11, and so on. The series associated with this is ∑n=1∞ a_n, where an is the n^{th} prime number. The partial sum up to 4 terms is 2+3+5+7=17.

RULES OF SEQUENCES

We can generally have two types of rules for a sequence (if it is geometric/arithmetic).

- Implicit rule where a term is expressed in terms of its previous term.
- Explicit rule where any term can be found using a general formula.

Example: Consider the sequence of odd numbers 3, 5, 7, …. We will define two rules to define n^{th} term (general term) of this sequence. Note that this sequence is an arithmetic sequence with the first term 3 ($a = 3$) and the common difference 2 ($d = 5 - 3 = 2$). Then:

Implicit rule: $a_n = a_{n-1} + 2$

Explicit rule: $a_n = a + (n - 1)\, d = 3 + (n - 1)\, 2 = 3 + 2n - 2 = 2n + 1$.

Here, $a_n = a + (n - 1)\, d$ is the formula for n^{th} term of the arithmetic sequence

SEQUENCES FORMULAS

As we have seen in the previous section, the formula for a sequence is nothing but the formula for its n^{th} term. Let us see the formulas for the n^{th} term (an) of different types of sequences in math.

Arithmetic sequence: $a_n = a + (n - 1) d$, where a = the first term and d = common difference.

Geometric sequence: $a_n = ar^{n-1}$, where a = the first term and r = common ratio.

Fibonacci sequence: $a_{n+2} = a_{n+1} + a_n$. The first two terms are 0 and 1.

Square number sequence: $a_n = n^2$.

Cube number sequence: $a_n = n^3$.

Triangular number sequence: $a_n = \sum k=1^{n}\ n$. This can be further evaluated

PRACTICE: SEQUENCE MAXIMUMS PROBLEM

Problem. You have the sequence A of the N natural numbers which are computed by formulas: A_1=constant; $A_i=(A_{i-1}*B+C)$ mod M.

You need to find K maximal numbers in the sequence.

Task. Create a program using C/C++/Python to solve this problem.

Input. A single string with six numbers N ($3<=N<=3*10^7$), K ($1<=K<=\min(200, N)$), B ($1<=B<=10^9$), C ($1<=C<=10^9$), M ($2<=M<=10^9$), and A_1 ($1<=A_1<M$) divided by spaces.

Output. A single string with K maximal numbers from sequence A in ascending order.

PRACTICE: SEQUENCE MAXIMUMS PROBLEM

Example: N=5, K=3, B=2, C=7, M=9, and A_1=1.

Input: 5 3 2 7 9 1

$A_1 = 1$

$A_2 = (A_1*B+C) \bmod M = (1*2+7) \bmod 9 = 0$

$A_3 = (A_2*B+C) \bmod M = (0*2+7) \bmod 9 = 7$

$A_4 = (A_3*B+C) \bmod M = (7*2+7) \bmod 9 = 3$

$A_5 = (A_4*B+C) \bmod M = (3*2+7) \bmod 9 = 4$

A is [1, 0, 7, 3, 4] then K maximal in ascending order are [3, 4, 7]

PRACTICE: SEQUENCE MAXIMUMS PROBLEM

Code example:

```
#include <iostream>
#define MAX 200
int N, K, B, C, M, A1, R[MAX];
int main()
{
    cin >> N >> K >> B >> C >> M >> A1;

    // Your code here

    for(int i = 0; i < K; i++)
    {
        cout << R[i] << " ";
    }
    return 0;
}
```

THANK YOU!