PLANAR CLIPPING

- Clipping lines and polygons
- Analytical clipping
- Cohen-Sutherland algorithm
- Liang-Barsky algorithm

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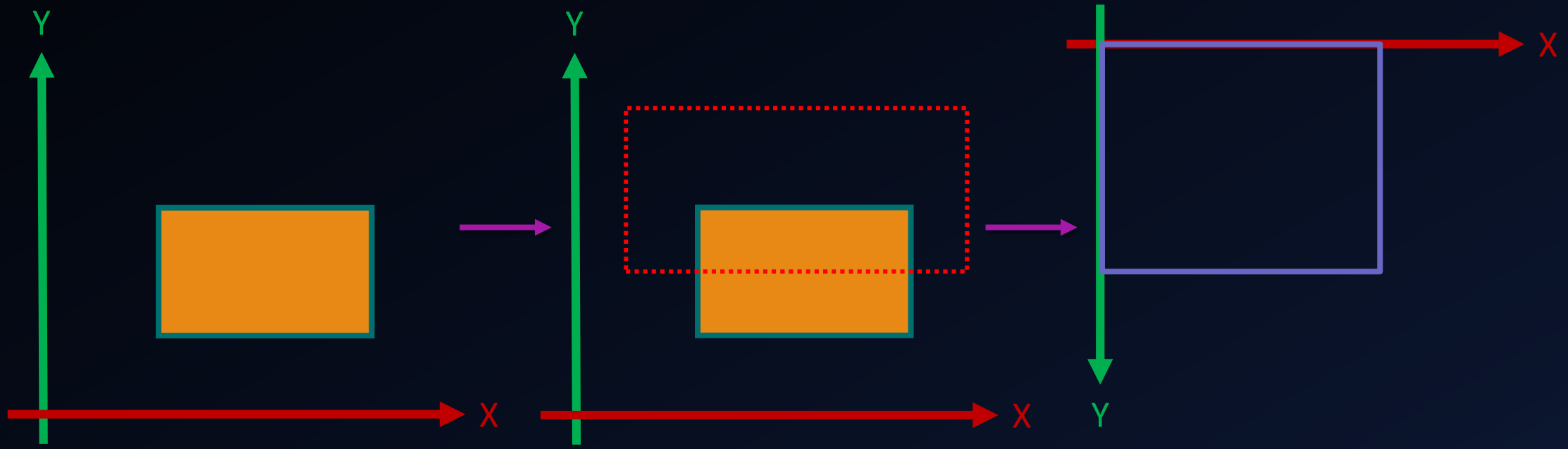
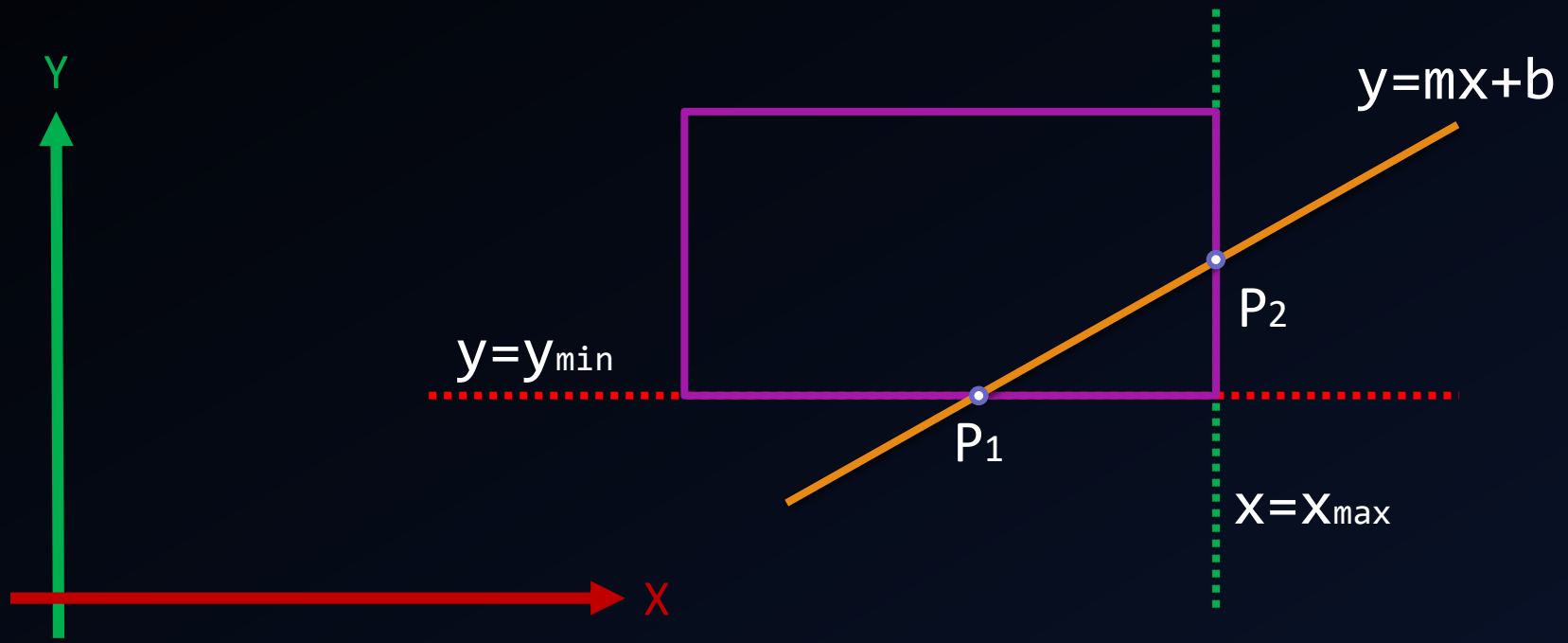
CLIPPING LINES AND POLYGONS

Goal: only render what will be visible in the drawing window; eliminate from the rendering pipeline all those primitives which will not appear in the final viewport.



Analytical clipping

Brute force method: compute all intersections of line with clip rectangle



$P_1 = \{y=y_{min}\} \cap \{y=mx+b\}$

$P_2 = \{x=x_{max}\} \cap \{y=mx+b\}$

Cohen-Sutherland algorithm

Use regions to accept/reject lines.
Assume the clipping region is an axis-oriented rectangle.
Regions are labeled according to the half-planes defined by the clipping region.

T	B	R	L

TOP : $y > y_{max}$
BOTTOM : $y < y_{min}$
RIGHT : $x > x_{max}$
LEFT : $x < x_{min}$

Cohen-Sutherland algorithm

Let c_1 and c_2 be the region codes for the endpoints of the line to be clipped.

Trivially Accept: both codes are **0000** then segment is inside the clip region.

Trivially Reject: (c_1 & c_2) are not equal to zero then line is outside clip rectangle.

Repeat until the segment cannot be trivially accepted or rejected:

- Pick the endpoint which is outside the clip rectangle (one must be outside)
- Find the first non-zero bit: this corresponds to the clip edge which intersects the line
- Compute the intersection of the line with the edge
- Throw away the outside vertex up to the clip rectangle

TOP

$$x = x_0 + (x_1-x_0)*(y_{max}-y_0)/(y_1-y_0)$$
$$y=y_{max}$$

LEFT

$$y = y_0 + (y_1-y_0)*(x_{min}-x_0)/(x_1-x_0)$$
$$x=x_{min}$$

BOTTOM

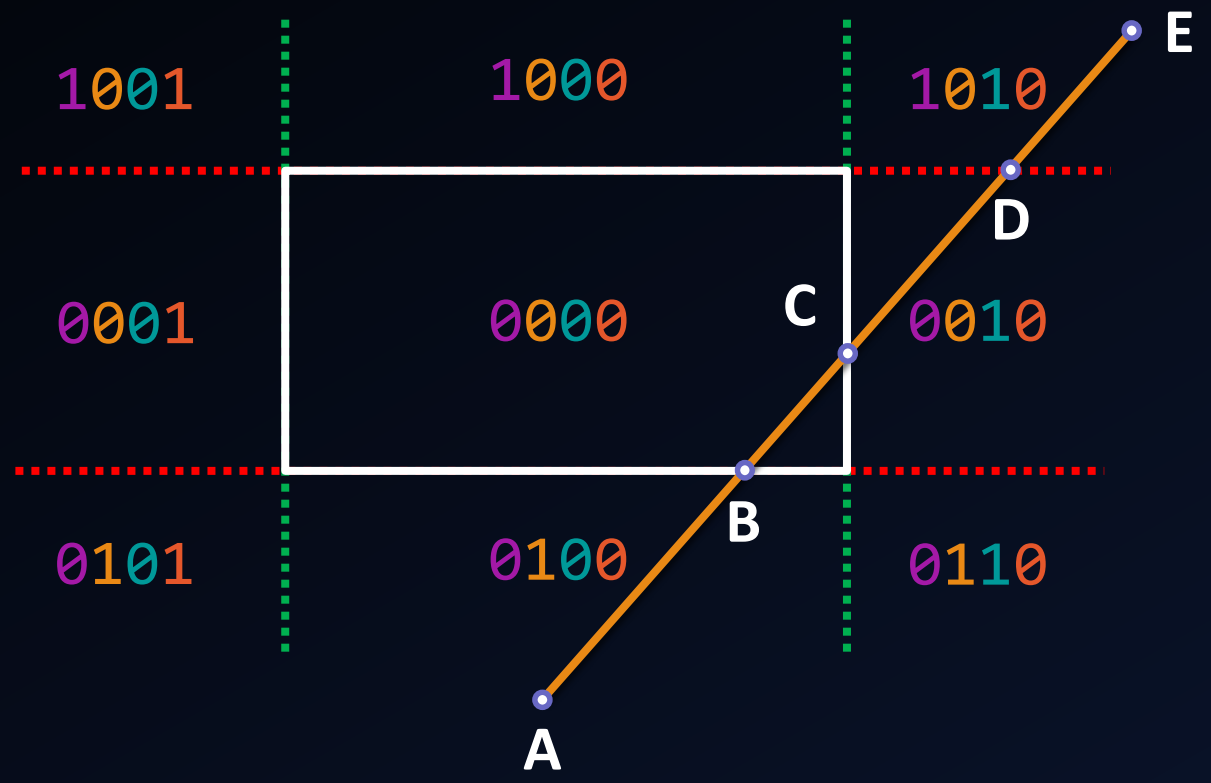
$$x = x_0 + (x_1-x_0)*(y_{min}-y_0)/(y_1-y_0)$$
$$y=y_{min}$$

RIGHT

$$y = y_0 + (y_1-y_0)*(x_{max}-x_0)/(x_1-x_0)$$
$$x=x_{max}$$

COHEN-SUTHERLAND ALGORITHM

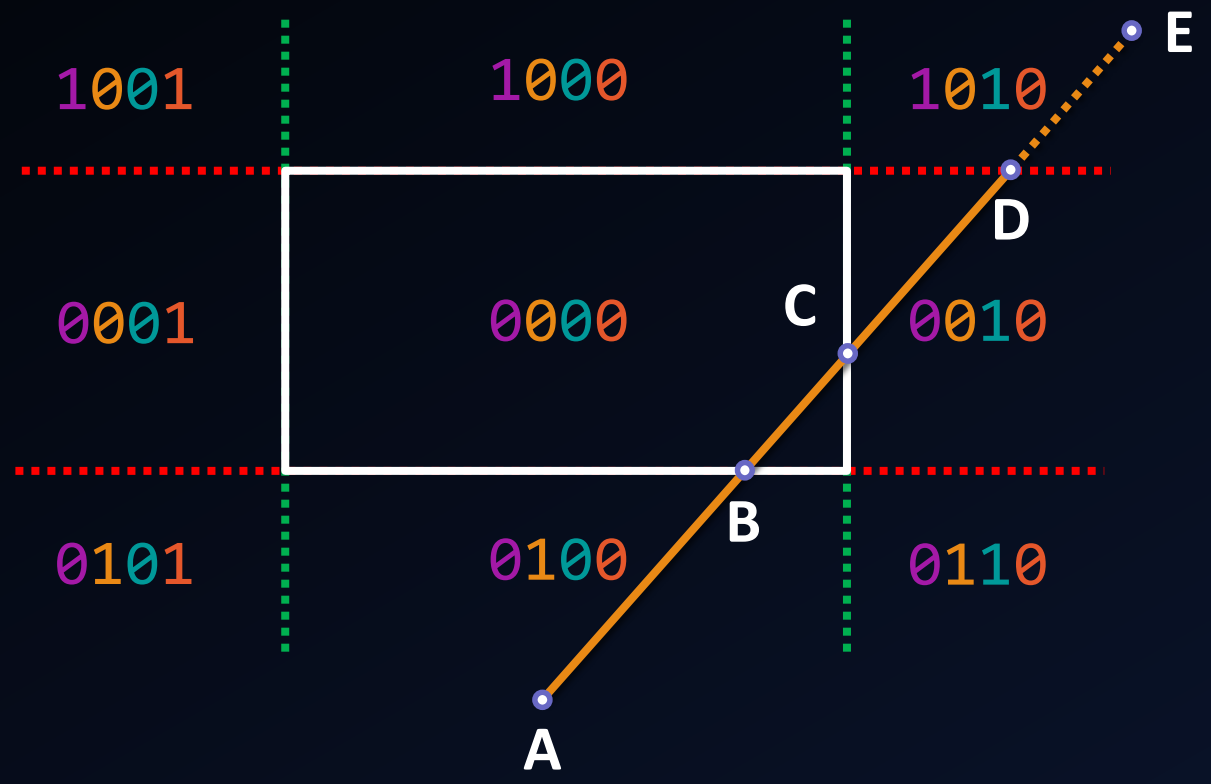
An example. STEP 1.



E : 1010
A : 0100 Subdivide using
& : 0000 either A or E

COHEN-SUTHERLAND ALGORITHM

An example. STEP 2.

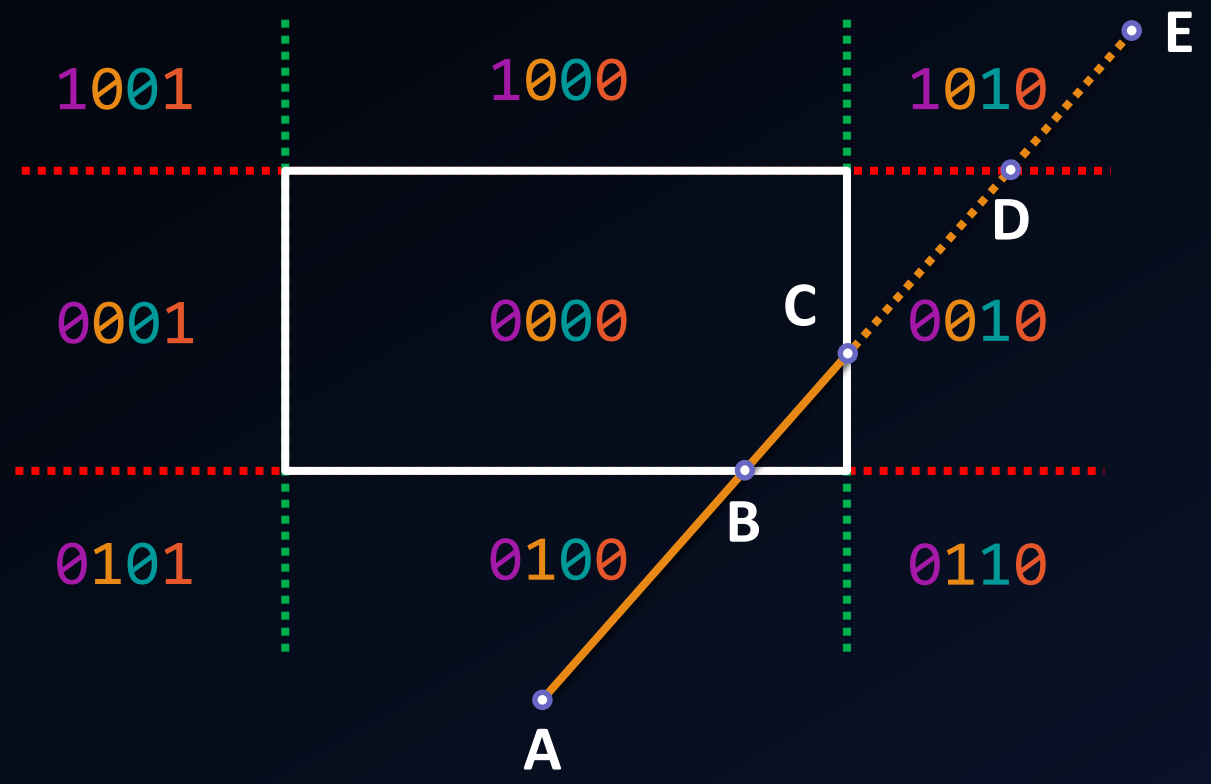


D : 0010
A : 0100
& : 0000

Subdivide: third bit of D indicates that the line intersects the right clip line

COHEN-SUTHERLAND ALGORITHM

An example. STEP 3.

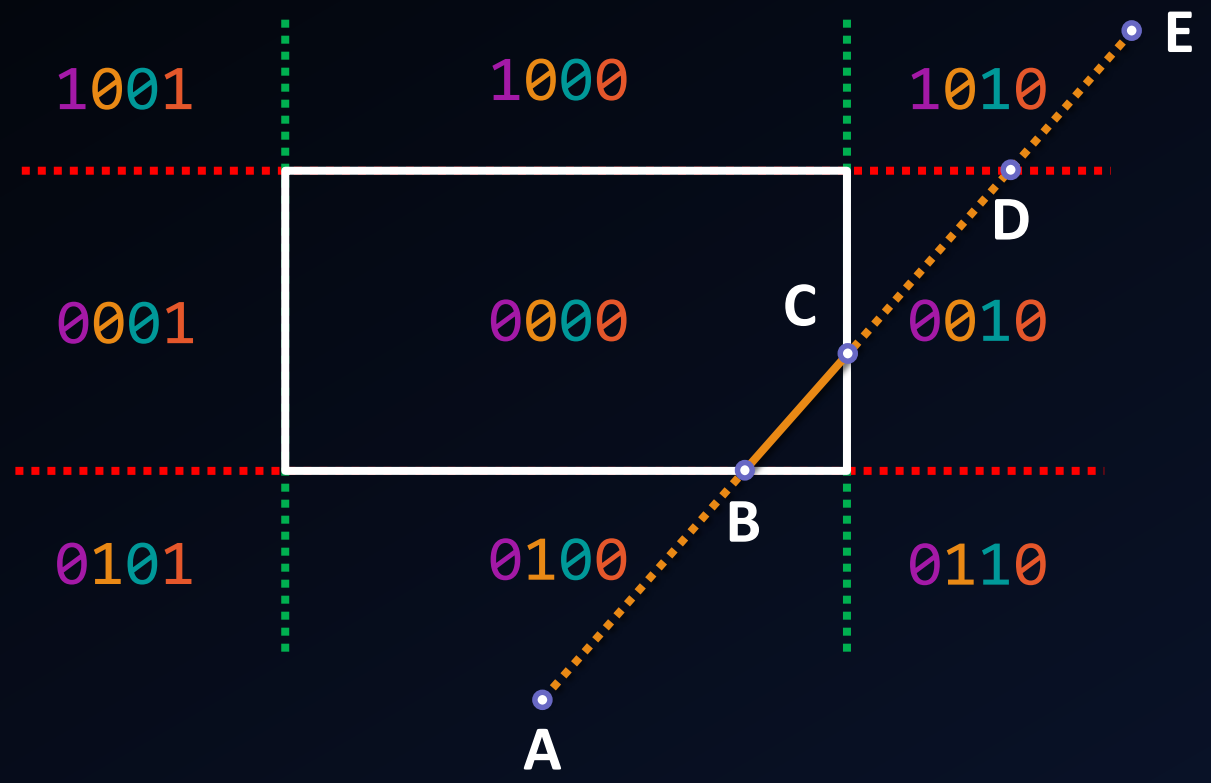


C : 0000
A : 0100
& : 0000

Subdivide: second bit of A indicates that the line intersects the bottom clip line

COHEN-SUTHERLAND ALGORITHM

An example. STEP 4.



C : 0000
B : 0000 Trivially accept the
& : 0000 clipped segment BC

Cohen-Sutherland algorithm

Disadvantages:

- Fixed-order decision can do needless work
- Can improve using more regions (Nicholl-Lee-Nicholl Algorithm)
- Can generate more efficient rejection tests
- Clipping window must be rectangular

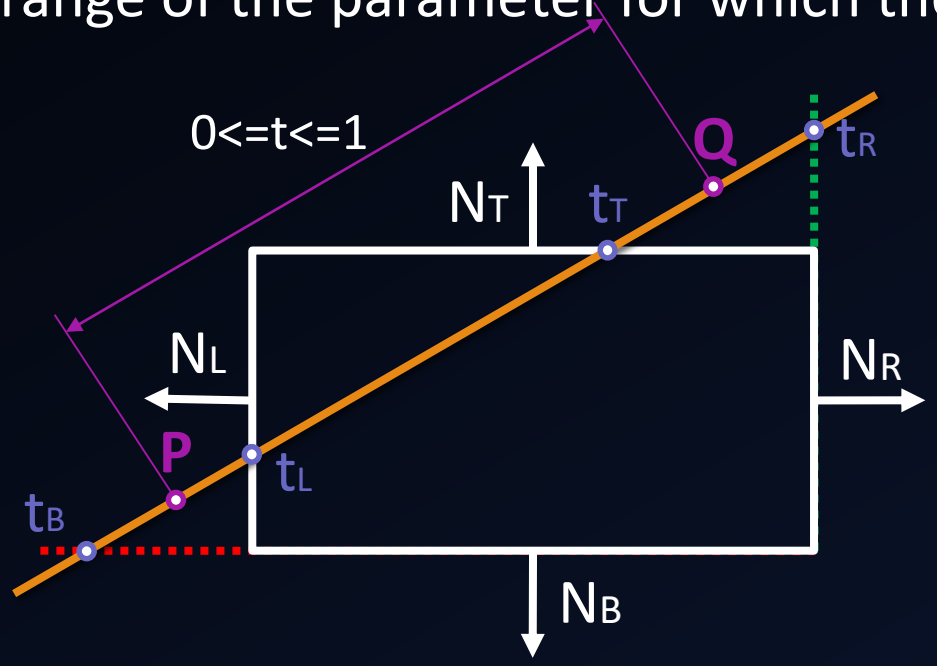
Advantages:

- Simple to implement
- Oriented for most simple window/viewport systems
- Extends to 3-D cubic volumes

LIANG-BARSKY ALGORITHM

The ideas for clipping line of Liang-Barsky and Cyrus-Beck are the same. The only difference is Liang-Barsky algorithm has been optimized for an upright rectangular clip window.

Liang and Barsky have created an algorithm that uses floating-point arithmetic but finds the appropriate end points with at most four computations. This algorithm uses the parametric equations for a line and solves four inequalities to find the range of the parameter for which the line is in the viewport.



Let **P**(x_1,y_1), **Q**(x_2,y_2) be the line which we want to study. The parametric equation of the line segment from gives x-values and y-values for every point in terms of a parameter that ranges from 0 to 1. The equations are:

$x=x_1+(x_2-x_1)*t=x_1+dx*t$

$y=y_1+(y_2-y_1)*t=y_1+dy*t$

LIANG-BARSKY ALGORITHM

The algorithm

1. Set $t_{min}=0$ and $t_{max}=1$
2. Calculate the values of t_L, t_R, t_T, and t_B

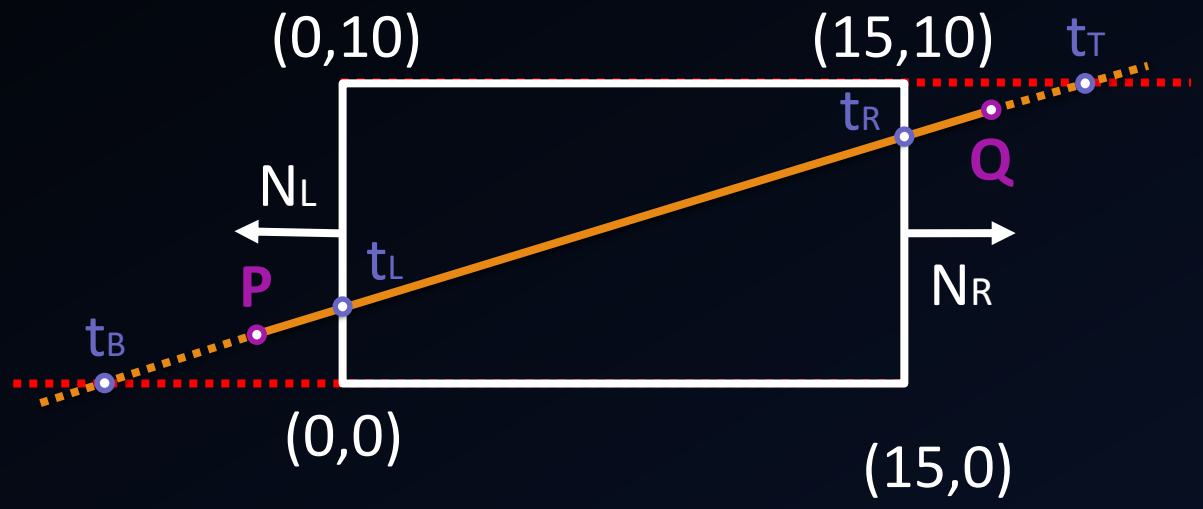
- if $t<t_{min}$ or $t>t_{max}$ ignore it and go to the next edge
- otherwise classify the **t** value as entering or exiting (using inner product with edge normals to classify)
- it **t** is entering value set $t_{min}=t$; it **t** is exiting value set $t_{max}=t$

3. If $t_{min}<t_{max}$ then draw a line from $(x_1+dx*t_{min}, y_1+dy*t_{min})$ to $(x_1+dx*t_{max}, y_1+dy*t_{max})$
4. If $t_{min}>t_{max}$ then segment out of the viewport and rejected

$$t_B = \frac{(B - y_1)}{(y_2 - y_1)} \qquad t_L = \frac{(L - x_1)}{(x_2 - x_1)} \qquad t_T = \frac{(T - y_1)}{(y_2 - y_1)} \qquad t_R = \frac{(R - x_1)}{(x_2 - x_1)}$$

LIANG-BARSKY ALGORITHM

The example #1. STEP 1. The **t** values computation.
B=0, **T**=10, **L**=0, **R**=15, **P**(-5,3), **Q**(20,9).



$$t_B = \frac{(0-3)}{(9-3)} = -\frac{1}{2}$$

$$t_L = \frac{(0-(-5))}{(20-(-5))} = \frac{1}{5}$$

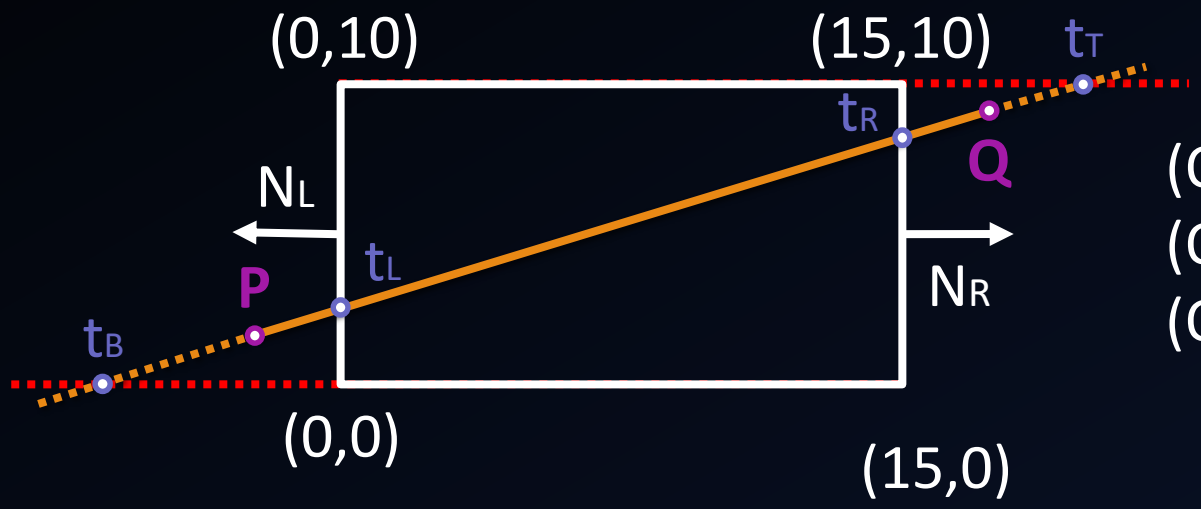
$$t_R = \frac{(15-(-5))}{(20-(-5))} = \frac{4}{5}$$

$$t_T = \frac{(10-3)}{(9-3)} = \frac{7}{6}$$

$t_B<0$ and $t_T>1$ are ignored
t_L and t_R are tested by inner product

Liang-Barsky algorithm

The example #1. STEP 2. The testing with edges normal.
B=0, **T**=10, **L**=0, **R**=15, **P**(-5,3), **Q**(20,9).

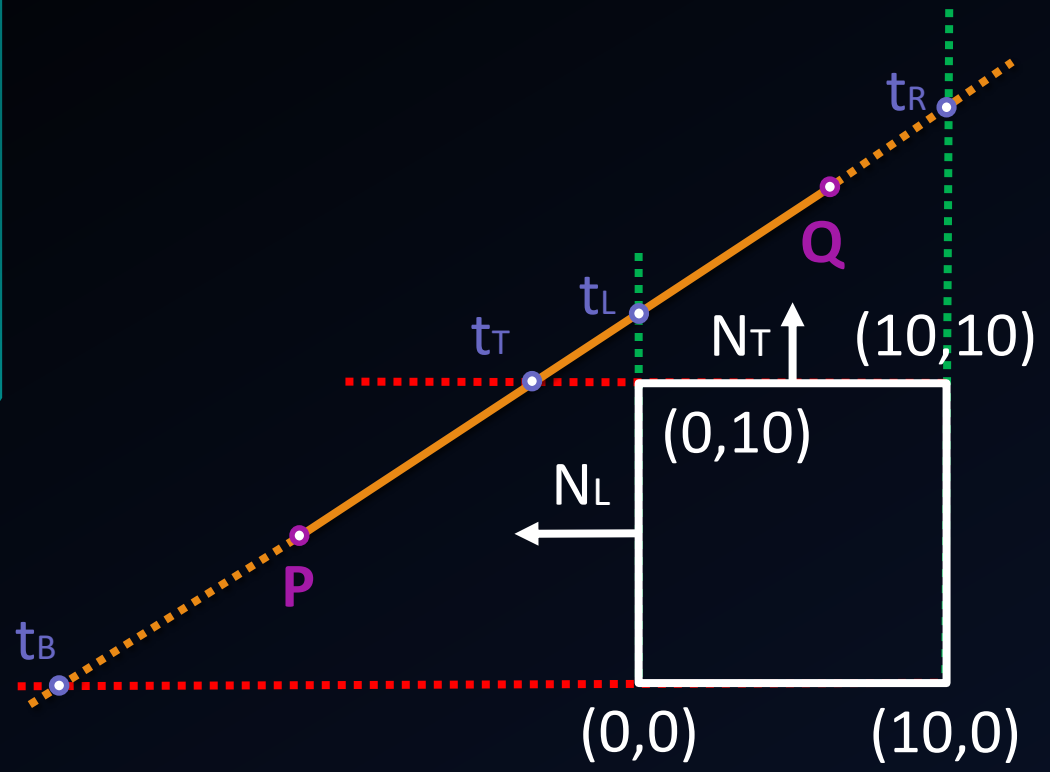


(Q-P)=(20+5,9-3)=(25,6)
(Q-P)*N_L=(25,6)*(-15,0)=-25<0, $\mathbf{t}_{min}=\mathbf{t}_L$
(Q-P)*N_R=(25,6)*(15,0)=25>0, $\mathbf{t}_{max}=\mathbf{t}_R$

Because $\mathbf{t}_{min}<\mathbf{t}_{max}$ we draw a line
from (-5+25*(1/5), 3+6*(1/5)) to (-5+25*(4/5), 3+6*(4/5))

LIANG-BARSKY ALGORITHM

The example #2. STEP 1. The **t** values computation.
B=0, **T**=10, **L**=0, **R**=10, **P**(-8,2), **Q**(2,14).



$$t_B = \frac{(0-2)}{(14-2)} = -\frac{1}{6}$$

$$t_T = \frac{(10-2)}{(14-2)} = \frac{8}{12}$$

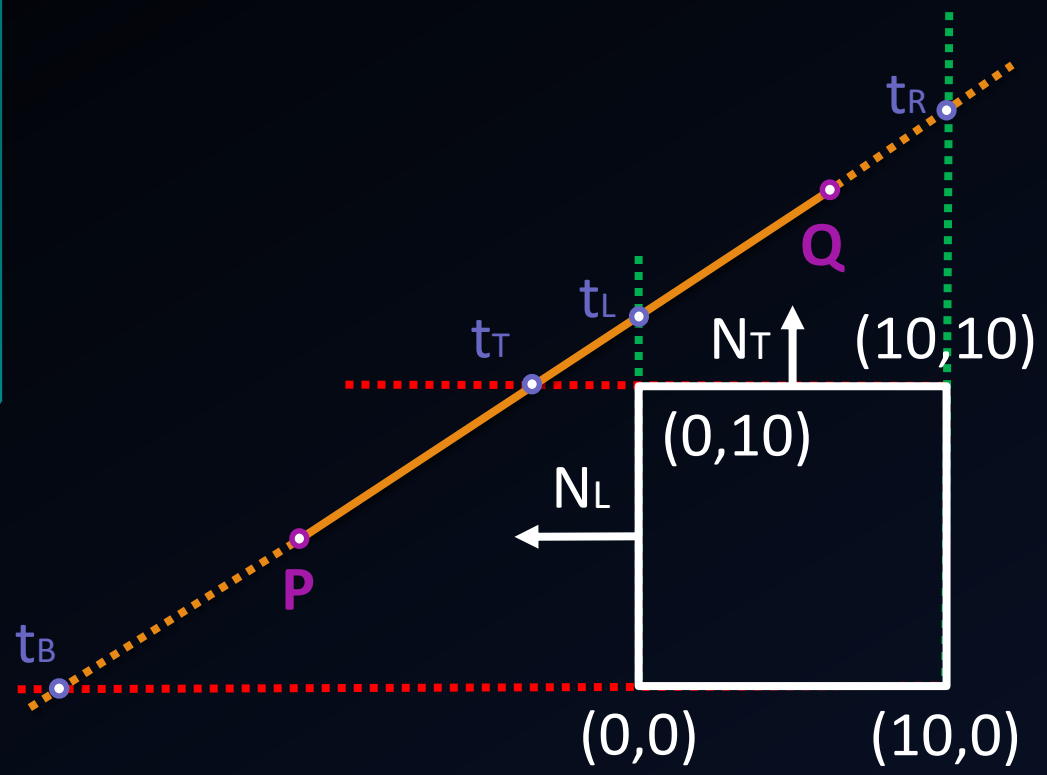
$$t_L = \frac{(0-(-8))}{(2-(-8))} = \frac{8}{10}$$

$$t_R = \frac{(10-(-8))}{(2-(-8))} = \frac{18}{10}$$

$t_B<0$ and $t_R>1$ are ignored
t_L and t_T are tested by inner product

LIANG-BARSKY ALGORITHM

The example #2. STEP 2. The testing with edges normal.
B=0, **T**=10, **L**=0, **R**=10, **P**(-8,2), **Q**(2,14).



(Q-P)=(2+8,14-2)=(10,12)
(Q-P)*N_T=(10,12)*(0,10)=120>0, $\mathbf{t}_{max}=\mathbf{t}_T$
(Q-P)*N_L=(10,12)*(-10,0)=-100<0, $\mathbf{t}_{min}=\mathbf{t}_L$

Because $\mathbf{t}_{min}>\mathbf{t}_{max}$ we don't have draw a line

Thank you!