Розрахункова робота
«Диференціальні рівняння»

1. – 3. Знайдіть загальний інтеграл диференціального рівняння.
4. Знайдіть розв’язок задачі Коші.
5. - 6. Знайдіть загальний інтеграл диференціального рівняння.
7. Знайдіть загальний інтеграл диференціального рівняння.
8 .- 9. Знайдіть загальний інтеграл диференціального рівняння.
10. Знайдіть загальний інтеграл диференціального рівняння.
11. Знайдіть розв’язок задачі Коші.
12. – 13. Знайдіть загальний розв’язок диференціального рівняння.
14. Знайдіть загальний розв’язок диференціального рівняння методом варіації довільної сталої.
15. Знайдіть розв’язок задачі Коші системи диференціальних рівнянь.

Варіант 1

1. $xy' = y^2 + 1;$
2. $(y^2 - 3x^2)dx + 2xydy = 0;$
3. $(2x-2)dy - (x+2y-3)dx = 0;$
4. $y' + \frac{y}{1+x} + x^2 = 0, \quad y(0) = 1;$
5. $y' + \frac{y}{x} = x^2 y^4;$
6. $(y + \frac{1}{1+x^2})dx + (x - \frac{1}{1+y^2})dy = 0;$
7. $\sqrt{1+x^2} \cdot y'' - 1 = 0;$
8. $xy'' = y' + x^3;$
9. $4y''\sqrt{y} = 1;$
10. $y''' + 3y'' + 2y = 0;$
11. $y'' - 4y' + 3y = 0, \quad y(0) = 1, y'(0) = 0;$
12. $y'' + 2y' + y = 3xe^{2x};$
13. $y'' - 2y' - 8y = e^x - 8\cos 2x;$
14. $y'' + y = \frac{1}{\cos^3 x};$
15. $\begin{cases} \frac{dx}{dt} = x - y, \\ \frac{dy}{dt} = -4x + y. \end{cases} \quad x(0) = 1, \ y(0) = -1.$

Варіант 2

1. $\sqrt{1-y^2}dx - ydy = 0;$
2. $xy' = y(1+\ln\frac{y}{x});$
3. $(x+2y+1)dx-(2x-3)\mathrm{d}y = 0;$
4. $y'ctgx - y = 2\cos^2 x \cdot ctgx, \quad y(\frac{\pi}{2}) = 0;$
5. $(1+x^2)y' = xy + x^2y^2;$
6. $(2x + ye^{xy})dx + (1+xe^{xy})dy = 0;$
7. $y^{IV} = x^3 + \sin 3x;$
8. $x^3y''+x^2y' = 1;$
9. $y''(1+y) = (y')^2 + y';$
10. $y^{IV} - 6y'''+9y' = 0;$
11. $y''-2y'+3y = 0, \quad y(0) = 0, \ y'(0) = 1;$
12. $y''+4y = x\cos 2x;$
13. $y''-2y'+5y = 4e^x \cdot \sin 2x + 6xe^{2x};$
14. $y''-y = \frac{e^x}{e^x+1};$
15. $\begin{cases} \frac{dx}{dt} = x - y, \\ \frac{dy}{dt} = x + y. \end{cases} \quad x(0) = 1, \ y(0) = 0.$

Варіант 3

1. $xy(1+x^2)y'=1+y^2;$
2. $y'=e^{-\frac{x}{y}}+\frac{y}{x};$
3. $(3y+3)dx-(2x+y-1)\mathrm{d}y=0;$
4. $y'-3x^2y-x^3=0, \quad y(0)=1;$
5. $y'-\frac{3}{x}y=-x^3y^2;$
6. $(x^2-5y)dx+(y^2-5x)dy=0;$
7. $y^{IV}=\cos x+x^2;$
8. $y''-2y'\cdot ctgx=\sin^3 x;$
9. $y\cdot y''-(y')^2+(y')^3=0;$
10. $y'''-5y''+6y'=0;$
11. $y''+2y'-8y=0, \quad y(0)=2, \ y'(0)=1;$
12. $y''+2y'+5y=7e^{-x}\sin 2x;$
13. $y''-4y'+4y=2xe^{2x}+\cos x;$
14. $y''+4y=\frac{1}{\sin^2 x};$
15. $\begin{cases} \frac{dx}{dt}=2x+y, \\ \frac{dy}{dt}=3x+4y. \end{cases} \quad x(0)=1, \ y(0)=2.$

Варіант 4

1. $x\sqrt{1-y^2}dx+\sqrt{1-x^2}dy=0;$

2. $xy'-y=\sqrt{4x^2+y^2}$;

3. $(x+8y-9)dx-(10x-y-9)\mathrm{d}y=0;$

4. $y'-\dfrac{2y}{1-x^2}=\dfrac{1-x}{(1+x)^3}, \quad y(0)=\dfrac{3}{2};$

5. $y'-xy=xy^3;$

6. $(\ln y-2x)dx+(\dfrac{x}{y}-2y)dy=0;$

7. $y^{IV}=3x^2+2\sin x;$

8. $y''-\dfrac{y'}{x}=x^2\ln x;$

9. $1+(y')^2=y\cdot y'';$

10. $y^{IV}-2y'''+y''=0;$

11. $y''-3y'+2y=0, \quad y(0)=-3, \ y'(0)=1;$

12. $y''+2y'+y=e^{-x}(x^2+6x);$

13. $y''-4y'+3y=4e^x+\cos 2x;$

14. $y''-y'=\dfrac{1}{e^x+1};$

15. $\begin{cases}\dfrac{dx}{dt}=x-y,\\ \dfrac{dy}{dt}=-4x+4y.\end{cases} \quad x(0)=2, \ y(0)=1.$

Варіант 5

1. $y - xy' = 2(1 + x^2 y')$;

2. $xy' = x\sin\frac{y}{x} + y$;

3. $(5x - 7y + 1)dy + (x + y - 1)\mathrm{dx} = 0$;

4. $(x^2 - 1)y' - xy = x^3 - x, \quad y(0) = 1$;

5. $xy' = 3y - x^4 y^2$;

6. $(2xye^{x^2} + \ln y)dx + (e^{x^2} + \frac{x}{y})dy = 0$;

7. $y'' = \ln x$;

8. $y'' tgx = y' + 1$;

9. $y''(1 + y) = 5(y')^2$;

10. $y^{IV} + y''' = 0$;

11. $y'' + 8y' + 16y = 0, \ y(0) = 2, \ y'(0) = -1$;

12. $y'' + y = (x + 3)\cos x$;

13. $y'' - 2y' - 8y = 2e^x - 8\cos 2x$;

14. $y'' + y = \frac{1}{\sin x}$;

15. $\begin{cases} \frac{dx}{dt} = x - y, \\ \frac{dy}{dt} = -4x + y. \end{cases}$ $\mathrm{x}(0) = 1, \mathrm{y}(0) = 3.$

Варіант 6

1. $y' - \frac{y}{x} = \frac{2}{x};$

2. $y' = \frac{x+8y}{8x+y};$

3. $(8y+10x-1)dx + (5y+7x-4)\mathrm{d}y = 0;$

4. $y' + 2xy = 3x^2 \cdot e^{-x^2}, \quad y(0) = 1;$

5. $y' + \frac{1}{\sin^2 x} y = y^2 \cdot \frac{ctgx}{\sin^2 x};$

6. $3e^y dx + (x^3 e^y - 1)dy = 0;$

7. $y'' \cdot \sin^3 x = \sin 2x;$

8. $y'' = (x + \frac{1}{x})y';$

9. $2yy'' = (y')^2;$

10. $y''' - y'' - 9y' + 9y = 0;$

11. $y'' + 4y' + 20y = 0, \ \ y(0) = -1, \ \ y'(0) = 1;$

12. $y'' - 6y' - 7y = 4xe^{-x};$

13. $y'' + 2y' + 5y = 7e^{-x}\cos 2x + 5e^{-x};$

14. $y'' + y = \frac{1}{\cos 2x};$

15. $\begin{cases} \frac{dx}{dt} = -2x + y, \\ \frac{dy}{dt} = -3x + 2y. \end{cases} \quad x(0) = 1, \ y(0) = 3.$

Варіант 7

1. $(xy + x^3 y)y' = 1 + y^2;$

2. $xyy' = x^2 + y^2;$

3. $(x + y + 2)dx + (y - x - 1)\mathrm{d}y = 0;$

4. $xy' - y = y^2 \cdot \sin x, \quad y(\frac{\pi}{2}) = \frac{\pi}{2};$

5. $y' + \frac{2}{x} y = x^2 y^2;$

6. $(3x^2 + \frac{2}{y}\cos\frac{2x}{y})dx - \frac{2x}{y^2}\cos\frac{2x}{y}dy = 0;$

7. $y'' = x \cdot \sin x;$

8. $y'' + y' tgx = \sin 2x;$

9. $y'' = \frac{y'}{\sqrt{y}};$

10. $y^{IV} - 3y''' + 3y'' - y' = 0;$

11. $y'' - 4y' + 13y = 0,\ y(0) = 1,\ y'(0) = 2;$

12. $y'' - y' = -9x;$

13. $y'' - 2y' + 10y = 8e^x \sin 3x + 9xe^{3x};$

14. $y'' - 4y' + 5y = \frac{e^{2x}}{\cos x};$

15. $\begin{cases} \frac{dx}{dt} = 2x + y, \\ \frac{dy}{dt} = -6x - 3y. \end{cases} \quad x(0) = 1,\ y(0) = 2.$

Варіант 8

1. $(x+xy)dy-(x^2+1)ydx=0;$

2. $xy'+xtg\dfrac{y}{x}=y;$

3. $(x+y-2)dx-(3x-y-2)\mathrm{dy}=0;$

4. $y'+2xy=x\cdot\ln x\cdot e^{-x^2},\quad y(1)=0;$

5. $y'\cos^2 x+y=y^2\cdot tgx;$

6. $(3x^2+4y^2)dx+(8xy+e^y)dy=0;$

7. $y'''=x\cdot\cos x;$

8. $(1-x^2)y''-xy'=0;$

9. $y''tgy=2(y')^2;$

10. $y'''+y''-y'-y=0;$

11. $y''-6y'+9y=0,\ \ y(0)=-3,\ \ y'(0)=1;$

12. $4y''+4y'+1=4x^2\cdot e^{\frac{x}{2}};$

13. $y''+6y'+10y=17e^{-3x}\cos x+15xe^{-3x};$

14. $y''+9y=\dfrac{1}{\sin 3x};$

15. $\begin{cases}\dfrac{dx}{dt}=7x-3y,\\ \dfrac{dy}{dt}=x+5y.\end{cases}\quad x(0)=1,\ y(0)=2.$

Варіант 9

1. $y' = (2x-1)ctgy;$

2. $xy' + y\ln\frac{y}{x} = 0;$

3. $(x+7y-8)dx - (9x-y-8)\mathrm{d}y = 0;$

4. $y' - yctgx = \frac{1}{\sin x}, \quad y(\frac{\pi}{2}) = 0;$

5. $y' + y = x\cdot\sqrt{y};$

6. $(2x-1-\frac{y}{x})dx - (2y-\frac{1}{x})dy = 0;$

7. $y''' = \frac{\sin 2x}{\cos^3 2x};$

8. $y'' = x - \frac{y'}{x};$

9. $y''y^2 = 1;$

10. $y''' - 64y' = 0;$

11. $y'' - 2y' + y = 0,\ y(0) = -1,\ \ y'(0) = 3;$

12. $y'' - 10y' + 25y = 4xe^{5x};$

13. $y'' - 4y' + 13y = 18e^{2x}\sin 3x + 19xe^{3x};$

14. $y'' + 2y' + 2y = \frac{e^{-x}}{\cos x};$

15. $\begin{cases} \frac{dx}{dt} = 8x - 3y, \\ \frac{dy}{dt} = 2x + y. \end{cases} \quad x(0) = 3,\ y(0) = 1.$

Варіант 10

1. $(1+e^x)ydy - e^y dx = 0;$

2. $y' = \frac{y}{x} + \sin\frac{y}{x};$

3. $(x+3y+3)dx - (9x-3y+4)\mathrm{d}y = 0;$

4. $xy' - y = x^2 \sin x, \quad y(\frac{\pi}{2}) = \frac{\pi}{2};$

5. $y' + 2y = y^2 e^x;$

6. $(y^2 + \frac{y}{cox^2 x})dx + (2xy + tgx)dy = 0;$

7. $y''' = \frac{\cos x}{\sin^3 x};$

8. $y'' tgx = y' + 1;$

9. $2y \cdot y'' = 1 + (y')^2;$

10. $y^{IV} + 2y'' + y = 0;$

11. $y'' + 4y' + 4y = 0, \ y(0) = 1, \ y'(0) = -2;$

12. $y'' + 3y' = 8x^2;$

13. $y'' + 6y' + 13y = 27e^{-3x}\cos 2x + 25xe^{2x};$

14. $y'' - 2y' + 2y = \frac{e^x}{\sin^2 x};$

15. $\begin{cases} \frac{dx}{dt} = 4x - y, \\ \frac{dy}{dt} = -x + 4y. \end{cases} \quad x(0) = 1, \ y(0) = 1.$

Варіант 11

1. $(y^2+3)dx-\frac{e^x}{x}ydy=0;$

2. $xyy'=x^2+y^2;$

3. $(x-2y+3)dx+(2x+y+2)\mathrm{d}y=0;$

4. $y'\cos^2 x+y=tgx, \quad y(0)=-1;$

5. $y'+y\cdot tgx=y^4\cos x;$

6. $(3x^2y+2y+3)dx+(x^3+2x+3y^2)dy=0;$

7. $y''=\frac{\sin 2x}{\cos^2 2x};$

8. $x^3y''+x^2y'=1;$

9. $y^3\cdot y''=1;$

10. $y^{IV}+4y''+4y=0;$

11. $y''+4y'-21y=0,\ y(0)=2,\ y'(0)=-2;$

12. $y''-4y=x^2+2x;$

13. $y''-2y'+17y=13e^x\sin 4x+14xe^{4x};$

14. $y''+2y'+y=\frac{e^{-x}}{x};$

15. $\begin{cases}\frac{dx}{dt}=3x+y,\\ \frac{dy}{dt}=x+3y.\end{cases}\quad x(0)=2,\ y(0)=3.$

Варіант 12

1. $(1+y^2)dx-(y+yx^2)dy=0;$

2. $(x-y)y'=2x+y;$

3. $(2x+y-2)dx-(2x+2y-1)\mathrm{d}y=0;$

4. $y'+\frac{1}{x}y=x, \quad y(1)=1;$

5. $xy'+2y+x^5y^3e^x=0;$

6. $(\sin 2x-2\cos(x+y))dx-2\cos(x+y)dy=0;$

7. $y''=x\cdot\sin x;$

8. $y''x\ln x=y';$

9. $y(1-\ln y)\cdot y''+(1+\ln y)(y')^2=0;$

10. $y^{IV}+8y''+16y=0;$

11. $y''+y'+y=0,\ y(0)=1,\ y'(0)=-1;$

12. $y''-8y'+16y=6xe^{4x};$

13. $y''-8y'+17y=17e^{4x}\cos x+18xe^x;$

14. $y''+4y=tg2x;$

15. $\begin{cases}\frac{dx}{dt}=2x+y,\\ \frac{dy}{dt}=-6x-3y;\end{cases}\quad x(0)=1,\ y(0)=2.$

Варіант 13

1. $y' = (3y + 2)tgx;$

2. $xy' + y\ln^2\frac{y}{x} = 0;$

3. $(x + 3y + 4)dx - (3x + y - 6)\mathrm{d}y = 0;$

4. $(1 + x^2)y' + y = arctgx, \quad y(0) = 1;$

5. $y' - \frac{y}{x-3} = \frac{y^2}{x-3};$

6. $(xy^2 + \frac{x}{y^2})dx + (x^2y - \frac{x^2}{y^3})dy = 0;$

7. $y'' = tg^2 3x;$

8. $y'' = y' + x;$

9. $2yy'' = 1 + (y')^2;$

10. $y''' + 4y'' + 3y' = 0;$

11. $y'' - y' - 2y = 0, \ y(0) = -2, \ y'(0) = 1;$

12. $y'' - 4y' + 3y = (x^2 + 2)e^{3x};$

13. $y'' - 4y' + 20y = 21e^{2x}\sin 4x + 22xe^{2x};$

14. $y'' - 4y' + 4y = \frac{e^{2x}}{x^3};$

15. $\begin{cases} \frac{dx}{dt} = 3x + y, \\ \frac{dy}{dt} = -3x + 2y. \end{cases} \quad x(0) = 1, \ y(0) = 3.$

Варіант 14

1. $2xyy' = 1 - x^2;$

2. $xy' \ln\frac{y}{x} = x + y \ln\frac{y}{x};$

3. $(x + 8y - 9)dx - (10x - y - 9)dy = 0;$

4. $y' + 3x^2 y = x^3 e^{-x^3}, \quad y(0) = 0;$

5. $2y' - \frac{x}{y} = \frac{xy}{x^2 - 1};$

6. $(\frac{1}{x^2} + \frac{3y^2}{x^4})dx - \frac{2y}{x^3}dy = 0;$

7. $y'' = (x + 1)e^{3x};$

8. $xy'' - y' = x^2 e^x;$

9. $y''(1 + y) = 5(y')^2;$

10. $y''' + 3y'' + 2y' = 0;$

11. $y'' + 8y' + 16 = 0, \quad y(0) = -1, \; y'(0) = 3;$

12. $y'' - 4y' + 4y = 8e^{2x};$

13. $y'' + 8y' + 20y = 25e^{-4x}\cos 2x + 26xe^{2x};$

14. $y'' - y' = e^{2x} \cdot \cos(e^x);$

15. $\begin{cases} \frac{dx}{dt} = 2x + y, \\ \frac{dy}{dt} = 3x + 4y. \end{cases}$ $x(0) = 2, y(0) = 2.$

Варіант 15

1. $(1+e^x)yy' = e^x;$

2. $xy' = y + 2x\operatorname{ctg}\frac{y}{x};$

3. $(2x+3y-1)dx - (5x+y-5)dy = 0;$

4. $xy' - y = x^2\cos x, \quad y(\frac{\pi}{2}) = \frac{\pi}{2};$

5. $xy' - 4y = 2x^2e^x;$

6. $\frac{y}{x^2}\cos\frac{y}{x}dx - (\frac{1}{x^2}\cos\frac{y}{x} + 2y)dy = 0;$

7. $y''' = \sin 3x + x^3;$

8. $xy'' = y'\ln x;$

9. $yy'' = (y')^2 - (y')^3;$

10. $2y^{IV} + 4y'' + 2y = 0;$

11. $y'' - 14y' + 49y = 0, \;\; y(0) = -2, \;\; y'(0) = 1;$

12. $y'' + 7y' + 12y = 5xe^{-4x};$

13. $y'' - 6y' + 25y = 29e^{3x}\sin 4x + 30xe^{3x};$

14. $y'' + 9y' = \frac{1}{\cos 3x};$

15. $\begin{cases} \frac{dx}{dt} = -2x + y, \\ \frac{dy}{dt} = -3x + 2y. \end{cases} \quad x(0) = -2, \;\; y(0) = -4.$

Варіант 16

1. $\sin x \cdot \mathrm{tg} y dx - \frac{dy}{\sin x} = 0;$

2. $y' = \frac{x+2y}{2x-y};$

3. $(x-3y-1)dx-(5x-y+1)dy=0;$

4. $y'\sin^2 x + y = \mathrm{ctg}x, \quad y(\frac{\pi}{2}) = 1;$

5. $xy^2y' = x^2 + y^3;$

6. $\frac{1+xy}{x^2y}dx + \frac{1-xy}{xy^2}dy = 0;$

7. $y'' = \frac{1}{x} + x;$

8. $y''x\ln x = 2y';$

9. $y''+(y')^2 = 2e^{-y};$

10. $y'''-y''+y'-y=0;$

11. $y''-6y'+10y=0,\ y(0)=2,\ y'(0)=-1;$

12. $y''-2y'+y=(x^2+1)e^x;$

13. $y''-2y'+26y = 37e^x\sin 5x + 38xe^{5x};$

14. $y''-y = \frac{2e^x}{e^x-1};$

15. $\begin{cases} \frac{dx}{dt} = x - y, \\ \frac{dy}{dt} = -4x + y. \end{cases} \quad x(0)=3, \quad y(0)=-2.$

Варіант 17

1. $3e^{x}\sin y dx+(1-e^{x})\cos y dy=0;$

2. $xyy'=2x^{2}+y^{2};$

3. $(x+4y-5)dx-(x-y-1)dy=0;$

4. $\sqrt{1-x^{2}}y'+y=y^{2}\arcsin x, \quad y(0)=1;$

5. $(x+1)(y'+y^{2})=-y;$

6. $\frac{dx}{y}-\frac{x+y^{2}}{y^{2}}dy=0;$

7. $y'''=e^{2x}+x^{2};$

8. $xy''=y'+xe^{x};$

9. $y''y+1=(y')^{2};$

10. $y'''-2y''+5y'=0;$

11. $y''-7y'+10y=0,\ \ y(0)=2,\ \ y'(0)=1;$

12. $y''-3y'+2y=2xe^{x};$

13. $y''-10y'+26y=41e^{5x}\cos x+42xe^{5x};$

14. $y''+2y'+2y=e^{-x}ctgx;$

15. $\begin{cases}\frac{dx}{dt}=6x-y,\\ \frac{dy}{dt}=3x+2y.\end{cases}$ $x(0)=0, y(0)=2.$

Варіант 18

1. $y' = \dfrac{e^{2x}}{\ln y};$

2. $x^2 y' = y^2 + xy + x^2;$

3. $(2x + 2y - 2)dx - (x + y + 8)dy = 0;$

4. $y' - \dfrac{2x}{1 + x^2} y = \operatorname{arctg}^2 x, \quad y(0) = 0;$

5. $y' x + y = -xy^2;$

6. $3x^2 e^y dx + (x^3 e^y - 1)dy = 0;$

7. $y''' = \cos 3x - x;$

8. $(x + 1)y'' - (x + 2)y' = e^x (1 + x)^2;$

9. $y'' = 2yy';$

10. $y''' - 3y'' + 2y' = 0;$

11. $y'' - y' - 2y = 0, \;\; y(0) = -3, \;\; y'(0) = 2;$

12. $y'' + y' = x^2 + 7x;$

13. $y'' - 4y' + 29y = 45e^{2x} \sin 5x + 46xe^{2x};$

14. $y'' - 2y' + 2y = \dfrac{e^x}{\sin x};$

15. $\begin{cases} \dfrac{dx}{dt} = -x + 8y, \\ \dfrac{dy}{dt} = x + y. \end{cases} \quad x(0) = 6, \; y(0) = 0.$

Варіант 19

1. $(xy^3+x)dx+(x^2y^2-y^2)dy=0;$

2. $(2x+y)y'=x+2y;$

3. $(-2x+3y+1)dx-(x+y-2)dy=0;$

4. $y'+3tg3x\cdot y=\sin 6x, \quad y(0)=\frac{1}{3};$

5. $y'-xy=-y^3e^{-x^2};$

6. $\frac{y}{x^2}dx-\frac{xy+1}{x}dy=0;$

7. $y''=x\sin 2x;$

8. $x(\ln x+2)y''=y';$

9. $y''-y'e^{-y}=(y')^2;$

10. $y^{IV}+3y'''+2y''=0;$

11. $y''+4y=0,\ \ y(0)=-1,\ \ y'(0)=1;$

12. $y''-6y'+9y=(x+6)e^{3x};$

13. $y''+4y'+29y=49e^{-2x}\cos 5x+50xe^{2x};$

14. $y''-y'=\frac{1}{1+e^x};$

15. $\begin{cases}\frac{dx}{dt}=2x+y,\\ \frac{dy}{dt}=-6x-3y.\end{cases}$ $x(0)=-2,\ y(0)=5.$

Варіант 20

1. $2x^2yy'+y^2=2;$

2. $y'=\frac{y}{x}+\sqrt{1-\frac{y^2}{x^2}};$

3. $(x+6y-3)dx-(8x-y-7)dy=0;$

4. $(1+x^2)y'+y=\operatorname{arctg}x, \quad y(0)=1;$

5. $xy'-y=2\sqrt{x^3y};$

6. $(xe^x+\frac{y}{x^2})dx-\frac{1}{x}dy=0;$

7. $y''=x\cos 3x;$

8. $x^2y''+xy'=1;$

9. $y''-(y')^2=0;$

10. $y'''-3y''+2y'=0;$

11. $y''+10y'+25y=0,\ y(0)=1,\ y'(0)=2;$

12. $y''-3y'-10y=6xe^{-2x};$

13. $y''-6y'+34y=53e^{3x}\sin 5x+54xe^{5x};$

14. $y''-2y'+y=\frac{e^x}{x^2};$

15. $\begin{cases}\frac{dx}{dt}=-2x-3y,\\ \frac{dy}{dt}=-x.\end{cases}$ $x(0)=4,\ y(0)=0.$

Варіант 21

1. $y' = e^{x} \cdot x \cdot (1 + y^{2})$;

2. $(x - 2y) \cdot y' = x + y$;

3. $(5x + 4y - 3)dx - (6x - 4y - 1)dy = 0$;

4. $y' + \frac{1}{\sqrt{1 - x^{2}}} y = \frac{\arcsin x}{\sqrt{1 - x^{2}}}, \quad y(0) = -1$;

5. $y' + xy = x^{3}y^{3}$;

6. $e^{y}dx + (\cos y + xe^{y})dy = 0$;

7. $y'' = \frac{\ln x}{x}$;

8. $xy'' + y' = \ln x$;

9. $y'' + 2(y')^{2} = 0$;

10. $y''' - 3y' + 2y = 0$;

11. $y'' - 13y' + 36y = 0, \ y(0) = -2, \ y'(0) = 1$;

12. $y'' + 12y' + 36y = 5xe^{-6x}$;

13. $y'' + 6y' + 34y = 57e^{-3x}\cos 5x + 58xe^{-3x}$;

14. $y'' + 4y = \operatorname{ctg} 2x$;

15. $\begin{cases} \frac{dx}{dt} = x - y, \\ \frac{dy}{dt} = -4x + 4y. \end{cases} \quad x(0) = 2, \ y(0) = 7.$

Варіант 22

1. $\mathrm{ctgx}\cdot\cos^2 \mathrm{ydx}+\sin^2 \mathrm{x}\cdot\mathrm{tgydy}=0;$

2. $\mathrm{x}\cdot\mathrm{y}'=\mathrm{y}+3\mathrm{x}\cdot\sin\frac{\mathrm{y}}{\mathrm{x}};$

3. $(2\mathrm{x}+\mathrm{y}-4)\mathrm{dx}-(3\mathrm{x}+5\mathrm{y}+1)\mathrm{dy}=0;$

4. $\mathrm{xy}'+\mathrm{y}=\ln\mathrm{x}+1,\quad \mathrm{y}(1)=4;$

5. $\mathrm{y}'=\frac{\mathrm{x}}{\mathrm{y}}\mathrm{e}^{2\mathrm{x}}+\mathrm{y};$

6. $(y^3+\cos x)dx+(3xy^2+e^y)dy=0;$

7. $\mathrm{y}''=\frac{\cos 2\mathrm{x}}{\sin^2 2\mathrm{x}};$

8. $\mathrm{y}''+\frac{2}{1-\mathrm{y}}(\mathrm{y}')^2=0;$

9. $\mathrm{yy}''+\mathrm{y}=(\mathrm{y}')^2;$

10. $\mathrm{y}^{\mathrm{IV}}+\mathrm{y}''=0;$

11. $y''+12y'+35y=0,\ \ y(0)=2,\ \ y'(0)=0;$

12. $\mathrm{y}''-4\mathrm{y}'+3\mathrm{y}=5\mathrm{xe}^{3\mathrm{x}};$

13. $y''-8y'+41y=61e^{4x}\sin 5x+62xe^{4x}\quad;$

14. $\mathrm{y}''+4\mathrm{y}'+4\mathrm{y}=\frac{\mathrm{e}^{-2\mathrm{x}}}{\mathrm{x}^3};$

15. $\begin{cases}\frac{dx}{dt}=-x-2y,\\ \frac{dy}{dt}=3x+4y.\end{cases}\quad \mathrm{x}(0)=3,\ \mathrm{y}(0)=-4.$

Варіант 23

1. $\sin x \cdot y' = y\cos x + 2\cos x;$

2. $xy' = y + y \cdot \ln\frac{y}{x};$

3. $(7x - y - 6)dx - (x + 5y - 6)dy = 0;$

4. $xy' - \frac{y}{x+1} = x, \quad y(0) = 1;$

5. $y' + 3y = e^{2x}y^2;$

6. $xe^{y^2}dx + (x^2ye^{y^2} + tg^2y)dy = 0;$

7. $y'' = \frac{(1+x)^2}{x(1+x^2)};$

8. $xy'' = y' + x^2;$

9. $(y')^2 + 2yy'' = 0;$

10. $y''' - 3y' - y' + 3y = 0;$

11. $y'' - 12y' + 11y = 0, \quad y(0) = 0, \ y'(0) = 1;$

12. $y'' - 14y' + 49y = 7e^{7x};$

13. $y'' + 8y' + 41y = 65e^{-4x}\cos 5x + 65xe^{-4x};$

14. $y'' - y' = e^{2x}\sin(e^x);$

15. $\begin{cases} \frac{dx}{dt} = -x + 8y, \\ \frac{dy}{dt} = x + y. \end{cases} \quad x(0) = -2, \ y(0) = 2.$

Варіант 24

1. $1+(1+y')e^{y}=0;$

2. $(3x+y)y'=x+3y;$

3. $(x+y+2)dx-(2x+1)dy=0;$

4. $y'\,tgx-y=1,\quad y(\frac{\pi}{2})=1;$

5. $x(x-1)y'+y^3=xy;$

6. $(5xy^2-x^3)dx+(5x^2y-y)dy=0;$

7. $y''=x\sin x\cos x;$

8. $(1+x^2)y''=2xy';$

9. $y''y=y^2+(y')^2;$

10. $y'''+y'=0;$

11. $16y''+8y'+y=0,\ \ y(0)=-1,\ \ y'(0)=1;$

12. $y''+8y'+16y=6xe^{-4x};$

13. $y''-2y'+37y=69e^{x}\sin 6x+70e^{x};$

14. $y''+y=tg^2x;$

15. $\begin{cases}\dfrac{dx}{dt}=x+2y,\\ \dfrac{dy}{dt}=4x+3y.\end{cases}\quad x(0)=3,\ y(0)=3.$

Варіант 25

1. $y'\operatorname{ctg}x + y = 2;$

2. $y' = \dfrac{2y^2 - xy}{xy - x^2};$

3. $(2x + 3y - 5)dx - (5x - 5)dy = 0;$

4. $y' + y = \dfrac{e^{-x}}{1 - x}, \quad y(0) = 1;$

5. $(1 - x^2)y' - xy = xy^2;$

6. $(\cos(x + y^2) + \sin x)dx + 2y\cos(x + y^2)dy = 0;$

7. $y'' = x + \cos 5x;$

8. $xy'' = y' + x^3;$

9. $y''y - (y')^2 = y^2;$

10. $y''' - 8y = 0;$

11. $y'' + 9y' + 8y = 0, \quad y(0) = 2, \quad y'(0) = 1;$

12. $y'' - 5y' + 8y = (x^2 + x)e^{2x};$

13. $y'' - 12y' + 37y = 73e^{6x}\cos x + 74xe^{6x};$

14. $y'' + 2y' + y = 3e^{-x}\sqrt{x + 1};$

15. $\begin{cases} \dfrac{dx}{dt} = 4x + 2y, \\ \dfrac{dy}{dt} = 4x + 6y. \end{cases} \quad x(0) = 4,\ y(0) = -1.$

Варіант 26

1. $y'\sin x = y\cdot \ln y;$

2. $(x^2 - xy)y' + y^2 = 0;$

3. $(4x + 3y - 1)dy - (5y + x + 5)dx = 0;$

4. $xy' - 2y = x^3\cdot \ln x, \quad y(0) = 1;$

5. $2x^3yy' + 3x^2y^2 + 1 = 0;$

6. $(\cos x - x\sin x)ydx + (x\cos x - 2y)dy = 0;$

7. $y'' = xe^{3x};$

8. $y'' - 2y'\operatorname{ctg}x = \sin^3 x;$

9. $y''\operatorname{tg}y = 2(y')^2;$

10. $y''' - 3y' - 2y = 0;$

11. $y'' - 16y = 0, \;\; y(0) = -1, \;\; y'(0) = 1;$

12. $y'' + 4y' - 5y = 4e^{-5x};$

13. $y'' - 4y' + 40y = 77e^{2x}\sin 6x + 78xe^{2x};$

14. $y'' + 2y' + 5y = \dfrac{e^{-x}}{\sin 2x};$

15. $\begin{cases} \dfrac{dx}{dt} = x + 4y, \\ \dfrac{dy}{dt} = x + y. \end{cases} \quad x(0) = 4,\ y(0) = 0.$

Варіант 27

1. $e^{y}\sin y dx + tgy dy = 0;$

2. $(y^2 - 3x^2)dy + 2xydx = 0;$

3. $(x + y + 2)dx - (2x + y - 4)dy = 0;$

4. $y' + 2xy = xe^{-x^2}, \quad y(0) = -1;$

5. $y' + \frac{1}{x}y = 2y^2;$

6. $(x^2 - 4xy - 2y^2)dx + (y^2 - 4xy - 2x^2)dy = 0;$

7. $y'' = x\ln x;$

8. $(x^2 + 1)y'' = 4x(y' - 1);$

9. $(y - 1)y'' = 2(y')^2;$

10. $9y^{IV} + 6y'' + y = 0;$

11. $25y'' - 10y' + y = 0, \quad y(0) = 2, \quad y'(0) = -1;$

12. $4y'' - 4y' + y = (x + 7)e^{\frac{x}{2}};$

13. $y'' + 4y' + 40y = 81e^{-2x}\cos 6x + 82xe^{-2x};$

14. $y'' - y' = e^{2x}\sqrt{1 - e^{2x}};$

15. $\begin{cases} \frac{dx}{dt} = 8x - 3y, \\ \frac{dy}{dt} = 2x + y. \end{cases} \quad x(0) = 4,\ y(0) = 3.$

Варіант 28

1. $(1+e^{3y})xdx = e^{3y}dy;$

2. $xy'-y+xe^{\frac{y}{x}} = 0;$

3. $(2x-y-3)dx+(x+2y+1)dy = 0;$

4. $y'-ytgx = \dfrac{1}{\cos^3 x}, \quad y(0)=3;$

5. $y'+x\sqrt[3]{y} = 3y;$

6. $xy^2dx + y(x^2+y^2)dy = 0;$

7. $y''' = e^{3x} + x^2;$

8. $x(y''+1)+y'=0;$

9. $yy''-2yy'\ln y-(y')^2 = 0;$

10. $y'''-8y''+15y' = 0;$

11. $y''+y'+2y = 0, \ y(0)=0, \ y'(0)=1;$

12. $y''-2y' = 3x^2+1;$

13. $y''-6y'+45y = 85e^{3x}\sin 6x + 86xe^{3x};$

14. $y''-3y'+2y = 1+\dfrac{1}{1+e^x};$

15. $\begin{cases} \dfrac{dx}{dt} = 3x-2y, \\ \dfrac{dy}{dt} = 2x+8y. \end{cases} \quad x(0)=3, \ y(0)=0.$

Варіант 29

1. $y - xy' = 3(1 + x^2 y')$;

2. $2x^3 y' = y(2x^2 - y^2)$;

3. $(x - 3y + 7)dx + (x + y - 5)dy = 0$;

4. $y' - \frac{y}{x} = x\cos x, \quad y(\frac{\pi}{2}) = 0$;

5. $xy' + y = y^2 \ln x$;

6. $2(3xy^2 + 2x^3)dx + 3(2x^2 y + y^2)dy = 0$;

7. $y''' = \frac{x^2}{e^x}$;

8. $xy'' = y' + x^3$;

9. $yy'' + y^2 - (y')^2 = 0$;

10. $y''' - 6y'' + 12y' - 8y = 0$;

11. $y'' + y' + 3y = 0, \quad y(0) = 2, \; y'(0) = 0$;

12. $y'' + 8y' = (x - 1)e^{2x}$;

13. $y'' + 6y' + 45y = 89e^{-3x}\cos 6x + 90xe^{-3x}$;

14. $y'' + 4y' + 4y = e^{-2x}\ln x$;

15. $\begin{cases} \frac{dx}{dt} = 3x + y, \\ \frac{dy}{dt} = x + 3y. \end{cases} \quad x(0) = 2, y(0) = 0.$

Варіант 30

1. $y'\sqrt{1-x^2}-\cos^2 y=0;$

2. $(y^2-2xy)dx-x^2dy=0;$

3. $(x-2y+4)dx+(x+3y-1)dy=0;$

4. $xy'+y=\frac{\ln x}{x},\quad y(1)=2;$

5. $(\frac{y^2}{x}-x^3)dx=ydy;$

6. $(\sin y+y\sin x+\frac{1}{x})dx+(x\cos y-\cos x+\frac{1}{y})dy=0;$

7. $y''=x^2\cos 2x;$

8. $xy''=y'+x^2;$

9. $(y')^2+yy''=0;$

10. $y'''-y''-8y'+12y=0;$

11. $y''+6y'+9y=0,\quad y(0)=-3,\ y'(0)=1;$

12. $y''+2y'-8y=(3x+1)e^{2x};$

13. $y''-8y'-52y=93e^{4x}\sin 6x+94xe^{4x};$

14. $y''+4y=\frac{1}{\sin 2x};$

15. $\begin{cases}\frac{dx}{dt}=x+4y,\\ \frac{dy}{dt}=2x+3y.\end{cases}\qquad x(0)=0,\ y(0)=3.$